SECOND MIDTERM

Problem 4:

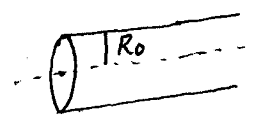
A very long cylinder of non-conductor and radius R_0 is charged with a volume charge density ρ given by $\rho = BR$ Between $R=0$ and $R=R_0$. B is constant.

(a) Calculate the magnitude of the electric field at any arbitrary distance R from the center (for $R<R_0$)

(b) Calculate the magnitude of the potential difference between $R=0$ and $R=R_0$.

(c) If the rod is negatively charged, what is the sign of $V(R_0)-V(0)$?

(d) Calculate the energy stored in the electric field for 1 meter length of this cylinder between $R=0$ and $R=R_0/2$.



Solution:

(a) Using Gauss law, we derive

$$\left.\begin{aligned} &\oint \vec{E}\cdot d\vec{S} = \frac{1}{\epsilon_0} q_{enclosed} \\ &q_{enclosed} = \int \rho dV = \int_0^R BR'\cdot 2\pi R' l dR' \\ &\oint \vec{E}\cdot d\vec{S} = E\cdot 2\pi R l \end{aligned}\right\} \Rightarrow E\cdot 2\pi R l = \frac{1}{\epsilon_0}\int_0^R 2\pi B l R'^2 dR'$$

$$\Longrightarrow E = \frac{1}{2\pi R l \epsilon_0}\cdot 2\pi B l \cdot \int_0^R R'^2 dR' = \frac{B}{\epsilon_0 R}\left(\frac{R'^3}{3}\Big|_0^R\right) = \frac{B}{\epsilon_0 R}\left(\frac{R^3}{3}-0\right) = \boxed{\frac{BR^2}{3\epsilon_0}}$$

(b) $$\Delta V = V(R_0) - V(0) = -\int_0^{R_0}\vec{E}\cdot d\vec{R} = -\int_0^{R_0}\frac{BR^2}{3\epsilon_0}dR = -\frac{BR_0^3}{9\epsilon_0}$$

The magnitude of the potential difference between $R=0$ and $R=R_0$: $|\Delta V| = \boxed{\frac{BR_0^3}{9\epsilon_0}}$

(c) If the rod is negatively charged, we know:

$\vec{E} = -\frac{BR^2}{3\epsilon_0}\hat{R}$ ($\hat{R}$ is unit vector)

$$\Delta V = V(R_0) - V(0) = -\int_0^{R_0}\vec{E}\cdot d\vec{R} = \int_0^{R_0}\frac{BR^2}{3\epsilon_0}dR = \frac{BR_0^3}{9\epsilon_0} > 0$$

$\boxed{\text{The sign is positive.}}$

(d) Total energy: $U = \int \frac{1}{2}\epsilon_0 E^2 dV = \frac{\epsilon_0}{2}\int_0^{\frac{R_0}{2}}\left(\frac{BR^2}{3\epsilon_0}\right)^2\cdot 2\pi R l dR = \frac{\pi B^2 l}{9\epsilon_0}\int_0^{\frac{R_0}{2}} R^5 dR$

$$\Rightarrow U = \frac{\pi B^2 l}{9\epsilon_0}\left(\frac{R^6}{6}\Big|_0^{\frac{R_0}{2}}\right) = \frac{\pi B^2 l}{9\epsilon_0}\left(\frac{1}{6}\left(\frac{R_0}{2}\right)^6 - 0\right) = \frac{\pi B^2 l}{54\epsilon_0}\frac{R_0^6}{2^6} = \boxed{\frac{\pi B^2 R_0^6}{3456\epsilon_0}}$$

$l = 1$ meter

Problem 4. Grading Scale.

For part (a), Write Gauss law, plus 2 points.
Write $\rho dV = BR \cdot 2\pi R \ell dR$, plus 2 points.
Write correct integral limit, plus 2 points
Write correct integral caculation, plus 2 points.
Get the final answer, plus 2 points.

For part (b) Write the formula $\Delta V = \int \vec{E} \cdot d\vec{R}$, plus two points
Write correct integral limits, plus two points
Get the final answer, plus one points.

For part (c) Get the final answer, plus 5 points.
Get the wrong answer, No points.

For part (d) Write $U = \int \frac{1}{2} \epsilon_0 E^2 dV$, plus 2 points.
Write $dV = 2\pi R \ell dR$, plus 2 points
Get the final answer, plus 1 points.