A cylindrical capacitor is created with the outer radius of the inner conductor \( R_A \) and the inner radius of the outer conductor \( R_B \).

(a) Calculate the capacitance (numerical value) of a 2.5 m length of this capacitor.

(b) What is the energy being stored between \( R = 0.50 \) cm and \( R = 1.00 \) cm in a 2.5 m length of this capacitor (numerical value). The capacitor has a potential difference of \( V = 175 \) V across it.

\[
R_A = 0.25 \text{ cm} \\
R_B = 1.25 \text{ cm}
\]

1. \[
C = \frac{\lambda}{4\pi V}
\]

2. \[
V_B - V_A = -\int_{R_A}^{R_B} E \cdot dr
\]

Using Gauss's law we can find \( E \):

3. \[
\int E \cdot A = \frac{Q}{\varepsilon_0} \implies 2\pi r l E = \frac{Q}{\varepsilon_0}
\]

\[
E = \frac{Q}{2\pi \varepsilon_0 l r} = \frac{1}{\varepsilon_0} \frac{2kQ}{l r} = \frac{E}{E}
\]

Substitute \( E \) to (2):

5. \[
V_B - V_A = -\int_{R_A}^{R_B} \frac{2kQ}{2\pi \varepsilon_0 l r} dr \\
= \frac{2kQ}{2\pi \varepsilon_0 l} \ln \frac{R_B}{R_A}
\]

Substituting into (1):

6. \[
C = \frac{\lambda}{4\pi V} = \frac{l}{2k \ln \frac{R_B}{R_A}}
\]

With numerical values:
\[
l = 2.5 \text{ m} \\
R_B = 1.25 \times 10^{-2} \text{ m} \\
R_A = 0.25 \times 10^{-2} \text{ m} \\
k = 9.00 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}
\]

\[
C = 8.63 \times 10^{-11} \left( \frac{\text{C}^2}{\text{Nm}} \right) \text{ or Farad } \approx 86.3 \text{ pF}
\]
Problem 4 MT 3 SP 1996

(b) \( U = \frac{1}{2} \varepsilon_0 \varepsilon^2 \) energy density in capacitor.

Since \( \varepsilon \) varies with \( r \) so energy density is not uniform inside the capacitor and we need integration

\[
(8) \quad dU = \frac{1}{2} \varepsilon_0 \varepsilon^2 dV
\]

In cylindrical coordinates \( dV = 2\pi r dl dr \)
and with \( \varepsilon \) from (4)

\[
\int dU = \int_{R_1}^{R_2} \frac{4kQ^2}{l^2} \frac{2\pi r dl dr}{r^2}
\]

\[
U = \frac{kQ^2}{l} \ln \frac{R_2}{R_1}
\]

(9)

\[
U = \frac{kQ^2}{l} \ln \frac{R_2}{R_1}
\]

the total charge for 2.5 m of this capacitor

\[
Q = CV = 8.63 \times 10^{-11} \times 175 V
\]

\[
Q = 1.51 \times 10^{-8} C
\]

Thus with the values for \( k \) and \( l \) and \( R_1 = 0.5 \text{cm} \)

\( R_2 = 1.0 \text{cm} \)

we can calculate total energy stored in 2.5 m of capacitor to be

\[
U = 5.69 \times 10^{-7} \text{J} \text{ or J}
\]