4

THIRD MIDTERM

Name (print)____________________ Name (signed)____________________

Discussion Instructor (circle): Gramada Hansen Li Zhukov

Discussion Section # ________

SHOW ALL WORK!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

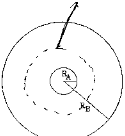
A cylindrical capacitor is created with the outer radius of the inner conductor R_A and the inner radius of the outer conductor R_B.

(a) Calculate the capacitance (numerical value) of a 2.5 m length of this capacitor.

(b) What is the energy being stored between R = 0.50 cm and R = 1.00 cm in a 2.5 m length of this capacitor (numerical value). The capacitor has a potential difference of V = 175 V across it.

$R_A = 0.25$ cm
$R_B = 1.25$ cm

Gauss's Surface



(1) $C = \frac{|Q|}{|V|}$

(2) $V_B - V_A = -\int_{R_A}^{R_B} E \cdot dr$

Using Gauss's law we can find E

(3) $\int \vec{E} \cdot \vec{n}\, da = \frac{Q}{\varepsilon_0} \Rightarrow 2\pi r l E = \frac{Q}{\varepsilon_0}$ → electric charge within the length l

$E = \frac{Q}{2\pi\varepsilon_0 l}\frac{1}{r} = \boxed{\frac{2kQ}{lr} = E}$ substitute E to (2)

(5) $V_B - V_A = -\int_{R_A}^{R_B} 2k\frac{Q}{l}\frac{dr}{r} \Rightarrow \boxed{V_B - V_A = -2k\frac{Q}{l}\ln\frac{R_B}{R_A}}$

substituting 5 into (1)

(6) $C = \frac{Q}{2k\frac{Q}{l}\ln\frac{R_B}{R_A}} \Rightarrow \boxed{C = \frac{l}{2k\ln\frac{R_B}{R_A}}}$

With numerical values $l = 2.5$ m $R_B = 1.25 \times 10^{-2}$ m $R_A = .25 \times 10^{-2}$ m

$k = 9.00 \times 10^{9} \frac{Nm^2}{C^2}$

$C = 8.63 \times 10^{-11} \frac{C^2}{N.m}$ or Farad $\boxed{86.3\ PF}$

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(7) b) $u = \frac{1}{2}\varepsilon_0 E^2$ energy density in capacitor,

Since E varies with r so energy density is not uniform inside the capacitor and we need integration

(8) $du = \frac{1}{2}\varepsilon_0 E^2 dV$

In cylinderical coordinates $dv = 2\pi r l\, dR$

and with E from (4)

$$\int du = \int_{R_1}^{R_2} \frac{1}{2}\varepsilon_0 \frac{4k^2Q^2}{l^2} \frac{2\pi l\, r dr}{r^2}$$

$$U = \frac{kQ^2}{l}\int_{R_1}^{R_2}\frac{dr}{r} = \frac{kQ^2}{l}\ln\frac{R_2}{R_1}$$

(9) $$\boxed{U = \frac{kQ^2}{l}\ln\frac{R_2}{R_1}}$$

the total charg for 2.5 m of this capacitor

$Q = CV = 8.63\times10^{-11} \times 175\,V$

$\boxed{Q = 1.51\times10^{-8}\,C}$

Then with the values for k and l and $R_1 = 0.5$ cm, $R_2 = 1.0$ cm we can calculate total energy stored in 2.5 m of capacitor to be

$\boxed{U = 5.69\times10^{-7}\,N.m}$ or J