Physics 172
Spring Quarter 1981
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FINAL EXAM

15

Name: ____

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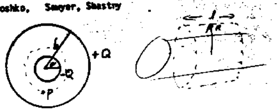

PROBLEM 5

Given two concentric, metal cylinders of length L and radii a and b. They are charged with charges +Q and -Q as shown.

a) Calculate the energy density at an arbitrary point P, a distance R from the center.

b) Set up, but do not evaluate, an integral by which one could calculate the total energy stored in this system.

(a) Use Gauss' Law: $\int \vec{E}\cdot d\vec{a} = \frac{q_{in}}{\epsilon_0} \Rightarrow E(2\pi R)l = \frac{\left(\frac{Q}{L}\right)l}{\epsilon_0} \Rightarrow E = \frac{Q}{2\pi\epsilon_0 RL} = \frac{2kQ}{RL}$ — E is radial, towards the axis

(10) $\Rightarrow E = \begin{cases} 0 & \text{outside cylinders or inside inner cylinder} \\ \frac{Q}{2\pi\epsilon_0 RL} & \text{between cylinders} \end{cases}$

(5) $\eta = \frac{1}{2}\epsilon_0 E^2 \Rightarrow \boxed{\eta = \begin{cases} 0 & \text{outside both cylinders or inside inner cylinder} \\ \frac{Q^2}{8\pi^2\epsilon_0 R^2L^2} & \text{between cylinders} \end{cases}}$

$\frac{kQ^2}{2\pi R^2L^2}$

(b) (10) $U = \int_{\text{all space}} \eta\, \underbrace{dV}_{L \cdot 2\pi R\, dR} = \int_a^b \frac{Q^2}{8\pi^2\epsilon_0 R^2L^2}(2\pi R\,dR)L \Rightarrow \boxed{U = \frac{Q^2}{4\pi\epsilon_0 L}\int_a^b \frac{dR}{R}}$

$\frac{kQ^2}{L}$

$\Rightarrow U = \frac{Q^2}{4\pi\epsilon_0 L}\ln\frac{b}{a}$

another way: this is a capacitor $\Rightarrow$

$\Rightarrow U = \frac{1}{2}QV = \frac{1}{2}Q\int \vec{E}\cdot d\vec{l} = \frac{1}{2}Q\int_a^b \frac{Q}{2\pi\epsilon_0 RL}dR = \frac{Q^2}{4\pi\epsilon_0 L}\int_a^b \frac{dR}{R}$