Williams

Physics 172
Winter Quarter 1986
February 21, 1986
George Williams

$\overline{X} = 13.1$
$\sigma = 5.31$
$n = 297$

THIRD MIDTERM

$\boxed{1}$

Grader ~~Name (Print)~~ Pollard Name (Signed)__________

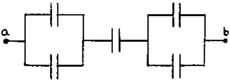
Discussion Instructor (Circle One): Brown Chung Pollard Rothman

Discussion Section #:________ Schweizer Soderberg Vaseghi Viohl

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

(a) Calculate the effective capacitance between a and b. All capacitors have a value of 350 pF.

$\boxed{175.0\ pF}$

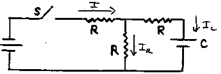


$\frac{1}{C_T} = \frac{1}{2C} + \frac{1}{C} + \frac{1}{2C} = \frac{2}{C} \rightarrow C_T = \frac{C}{2}$

(b) In the circuit shown, the switch is closed for a long time. Calculate the voltage across the capacitor 250 s after the switch is opened.

$\boxed{66.6\ Volts}$ See Solution on next page

$\epsilon = 150$ V
R = 350 Ω
C = 3.00 F

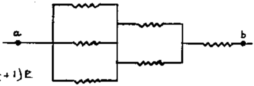


(c) Silicon has an atomic mass of 28.0 and a density of 2.40×10^3 kg/m^3. If a sample of silicon has a charge carrier density of 3.50×10^{16} cm^{-3}, how many carriers per atom does it have?

$\left(28\,\frac{gm}{mole}\right)\left(\frac{1\,m^3}{2.4\times10^6\,gm}\right)\left(\frac{1\,mole}{6.02\times10^{23}\,atoms}\right)\left(\frac{10^2\,cm}{m}\right)^3\left(3.50\times10^{16}\,\frac{carriers}{cm^3}\right) = \boxed{6.78\times10^{-7}\,\frac{carriers}{atom}}$

(d) Calculate the effective resistance between points a and b. All resistances are 7.00 ohm.

$\boxed{12.83\,\Omega}$

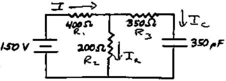


$R_T = \frac{1}{\frac{1}{R}+\frac{1}{R}+\frac{1}{R}} + \frac{1}{\frac{1}{R}+\frac{1}{R}} + R = \left(\frac{1}{3}+\frac{1}{2}+1\right)R = \frac{11}{6}R$

(e) Calculate the voltage across the capacitor at $t = \infty$.

$\boxed{50.0\,V}$

See Solution on next page



Front page

Solutions to b & e

b) This can be done by realizing that the voltage across the capacitor as $t \to \infty$ is $\frac{\mathcal{E}}{2}$ or 75.00 Volts and, for the discharging circuit, that $\tau = 2RC$.

Then, $V(t = 250s) = 75\, e^{-250/2100} = 66.58$ Volts.

It can also be done by the "uninspired" method. To do that, write down (i) junction equations, (ii) loop equations, and (iii) $I_c = \frac{dQ}{dt}$, Then solve the resulting differential equation.

CHARGING CIRCUIT

$$\left.\begin{aligned} I &= I_R + I_c \ , \quad I_c = \frac{dQ}{dt} \\ I_R R - I_c R - Q/_C &= 0 \\ \mathcal{E} - I R - I_R R &= 0 \end{aligned}\right\} \text{ for all } t \text{ in charging circuit}$$

putting together, we get

$$\frac{dQ}{dt} + \frac{2Q}{CR} = \frac{\mathcal{E}}{R}$$

the solution is

$$Q(t) = \left[Q(t{=}\infty) - Q(t{=}0)\right]\left[1 - e^{-t/\tau}\right] + Q(t = 0)$$

But we still need to calculate $Q(t{=}\infty)$ and $Q(t{=}0)$!

Note that $Q(t{=}\infty)$ will allow us to calculate what the voltage across the capacitor is when the switch is opened. We don't need the above D.E.!!

For $Q(t{=}\infty)$ note that $I_c \to 0$

We then have

$$\left.\begin{aligned} I &= I_R \ , \quad \frac{dQ}{dt} = 0 \\ I_R R - Q/_C &= 0 \\ \mathcal{E} - I R - I_R R &= 0 \end{aligned}\right\} \text{ for } t = \infty \text{ in charging circuit}$$

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b cont) We then have

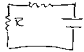
$$\mathcal{E} - I(R+R) = 0 \rightarrow I = \frac{\mathcal{E}}{2R}$$

$$IR - Q/C = 0 \rightarrow \frac{\mathcal{E}}{2} = \frac{Q}{C}$$

So the voltage across the capacitor as $t \to \infty$ is

$$V = \frac{Q}{C} = \frac{\mathcal{E}}{2} = 75.00 \text{ Volts}$$

DISCHARGING CIRCUIT (the switch is opened)



no junction eq's.

$$I(R+R) + Q/C = 0$$

$$I = \frac{dQ}{dt}$$

$$\rightarrow \frac{dQ}{dt} + \frac{Q}{2RC} = 0 \quad \therefore \tau = 2RC$$

the solution to this DE is $Q(t) = [Q(t=0) - Q(t=\infty)]e^{-t/\tau} + Q(t=\infty)$

here $Q(t=\infty) = 0$ & $Q(t=0)$ is the charge above

Since $V = \frac{Q}{C}$; $V(t) = \frac{Q(t=\infty)}{C} e^{-t/\tau} = (75.00 \text{ Volts}) e^{-t/\tau}$

So, $$\underline{\underline{V(t = 250 \text{ sec}) = (75.00)\, e^{-250/2100} \text{ Volts} = 66.58 \text{ Volts}}}$$

e) This can be done easily by realizing that for $t \to \infty$, the voltage across R_2 is equal to the voltage across the capacitor.

for $t \to \infty$, $V_{R_2} = I R_2 = \mathcal{E} \frac{R_2}{R_1 + R_2} = \mathcal{E}\left(\frac{1}{3}\right) = 50.0 \text{ Volts}$

$$\rightarrow V_C(t \to \infty) = 50.0 \text{ Volts}$$

The long way is just like charging circuit in part b), the difference being in the time constant as $R_1 \neq R_2 \neq R_3$

$$I = I_R + I_C \xrightarrow{t \to \infty} I = I_R = \frac{\mathcal{E}}{R_1 + R_2}$$

$$I_R R_2 - I_C R_3 - Q/C = 0 \xrightarrow{t \to \infty} I_R R_2 = Q/C$$

Note: We don't need to calculate τ !

$$\frac{Q(t)}{C} = V(t) = [V(t=\infty)]\left(1 - e^{-t/\tau}\right) \xrightarrow{t \to \infty} V(t=\infty) = \frac{Q(t=\infty)}{C} = I_R R_2$$

$$V_C(t=\infty) = \frac{\mathcal{E}}{R_1 + R_2} R_2 = \mathcal{E}\,\frac{200}{600} = \frac{1}{3}\mathcal{E} = 50.0 \text{ Volts}$$