Physics 302
Spring Quarter 1990
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George Williams

$\bar{X} = 9.08$
$\sigma = 13.37$
$N = 155$

4

THIRD MIDTERM

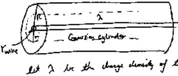
Name (print) MARK REEVE   Name (signed) Mark D Reeve

Discussion Instructor (circle one): Baselgia Morrill Reeve Stoops Zhang

Discussion Section #________

SHOW ALL WORK!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

A wire of radius r = 0.100 mm is at the center of a conducting cylinder of inner radius R = 1.25 cm. There is a potential difference of 525 V between the wire and the cylinder. Calculate the energy stored in a 2.00 m length of this structure between r and R/2.



$R = 1.25\times10^{-2}$ m
$r_w = 1\times10^{-4}$ m

Steps to Solve
① Calculate $E(\lambda)$
② Integrate to get $V \Rightarrow \lambda$.
So now we know E.
③ $u = \frac{1}{2}\epsilon_0 E^2$
④ $U = \int u\,dv$

Let λ be the charge density of the wire in C/m.
Then Gauss's law $\Rightarrow$ $E\cdot 2\pi r l = \frac{\lambda l}{\epsilon_0}$ $\Rightarrow$ $\underline{\vec{E} = \frac{\lambda}{2\pi r\epsilon_0}\hat{r}}$ $\boxed{5\text{pts}}$

$V = -\int \vec{E}\cdot d\vec{s}$ $\therefore$ $|V(R) - V(r_w)| = \left|-\int_{r_w}^{R}\frac{\lambda}{2\pi r\epsilon_0}dr\right| = \left|\frac{\lambda}{2\pi\epsilon_0}\ln\frac{r_w}{R}\right| = 525\text{ V}$

$\Rightarrow \lambda = \dfrac{(525\text{ V})(2\pi\times 8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}{\left|\ln\frac{1\times10^{-4}\text{ m}}{1.25\times10^{-2}\text{ m}}\right|} = \underline{6.046\times10^{-9}\ \text{C/m} = \lambda}$ $\boxed{7\text{pts}}$

$u = \frac{1}{2}\epsilon_0 E^2$
$U = \int u\,dv = \int_{r_w}^{R/2}\frac{1}{2}\epsilon_0\frac{\lambda^2}{(2\pi r\epsilon_0)^2}\cdot 2\pi r l\,dr = \frac{\lambda^2 l}{4\pi\epsilon_0}\int_{r_w}^{R/2}\frac{dr}{r} = \frac{\lambda^2 l}{4\pi\epsilon_0}\ln\frac{R}{2r_w}$ $\boxed{10\text{pts}}$

Plugging in the numbers with $\lambda = 6.046\times10^{-9}$ C/m and $l = 2.00$ m $\Rightarrow$ $\boxed{U = 2.72\times10^{-6}\text{ J} = \text{energy stored between } r_w \text{ and } R/2}$ $\boxed{3\text{pts}}$

COMMENTS: Most people had no clue how to do this problem — only two did it correctly. A common error was to assume the electric field to be constant. It isn't. Several people tried to use capacitance to do the problem. This is how it should go in that case:

Suppose you recall $U = \frac{1}{2}CV^2$ (or $\frac{Q^2}{2C}$ or $\frac{1}{2}QV$) is the energy stored in a capacitor.
First calculate (or read off your card) the capacitance of two concentric cylinders:

$E = \frac{\lambda}{2\pi r\epsilon_0}$ $\Rightarrow$ $V = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{r_w}{R}$ $\Rightarrow$ $C = \frac{Q}{V} = \frac{\lambda l}{V} = \frac{2\pi\epsilon_0 l}{\ln\frac{r_w}{R}} = C(R)$

Then $V(R) = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{r_w}{R} = 525\text{ V}$ $\Rightarrow$ $\lambda = \frac{525\cdot 2\pi\epsilon_0}{\ln\frac{r_w}{R}}$ $\Rightarrow$ $V(r) = \frac{525\ln\frac{r_w}{r}}{\ln\frac{r_w}{R}}$

$\therefore U = \frac{1}{2}CV^2 = \frac{1}{2}\underbrace{C(R/2)V(R/2)^2}_{\text{both change}} = \frac{1}{2}\left[\frac{2\pi\epsilon_0 l}{\ln\frac{2r_w}{R}}\right]\left[\frac{525\ln\frac{2r_w}{R}}{\ln\frac{r_w}{R}}\right]^2 = 2.72\times10^{-6}\text{ J}$ as above.