Physics 302
Winter Quarter 1992
March 17, 1992
George Williams

Avg = 14.3

6

FINAL EXAM

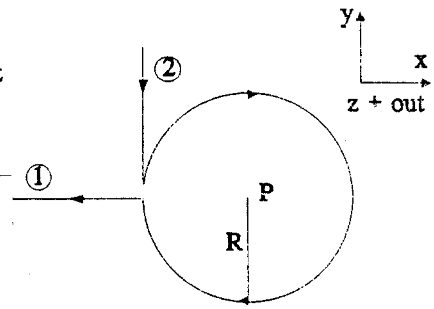
Name (print) J-K Leong   Name (signed) ____________

Discussion Instructor (circle): Condella   Godfrey-Smith   Guilkey   Leong   Nott   Paul

Discussion Section # ______

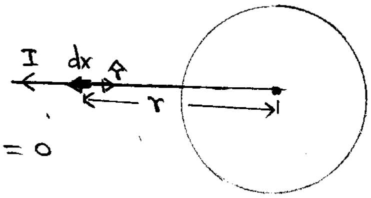
SHOW ALL WORK!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

Use the Biot-Savart law to calculate the magnetic field at point P (in $\hat{x}$, $\hat{y}$, $\hat{z}$ notation), due to a current of 5.00 amperes in the direction shown by the arrows. The round portion of the wire is circular (R = 6.00 cm), and P is at the center. For ease in grading, label the infinite straight segments (1) and (2) as shown in the figure.



Segment (1)

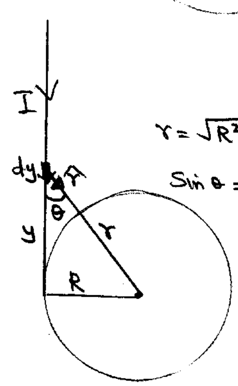
$$d\vec{B}_1 = \frac{\mu_0}{4\pi}\frac{I\,d\vec{s}\times\hat{r}}{r^2}$$



$$d\vec{s}\times\hat{r} = (dx)\sin 180^\circ = 0$$

$$\therefore \vec{B}_1 = 0$$

Segment (2)



$$r = \sqrt{R^2+y^2}$$

$$\sin\theta = \frac{R}{\sqrt{R^2+y^2}}$$

$$d\vec{B}_2 = \frac{\mu_0}{4\pi}\frac{I\,d\vec{s}\times\hat{r}}{r^2}$$

$$= \frac{\mu_0}{4\pi}\frac{I\,dy\sin\theta}{r^2}$$

$$\therefore |\vec{B}_2| = \frac{\mu_0 I}{4\pi}\int_0^\infty \frac{R}{(R^2+y^2)^{3/2}}\,dy$$

$$= \frac{\mu_0 I R}{4\pi}\left[\frac{y}{R^2\sqrt{R^2+y^2}}\right]\Bigg|_0^\infty$$

$$= \frac{\mu_0 I}{4\pi R}\left[\frac{1}{\sqrt{\left(\frac{R}{y}\right)^2+1}}\right]\Bigg|_0^\infty$$

$$= \frac{\mu_0 I}{4\pi R}$$

$$\therefore \vec{B}_2 = \frac{\mu_0 I}{4\pi R}(\hat{z})$$

Grading System

Segment (1) 9 pts
No reason (−3)

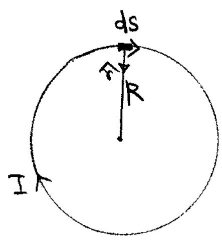
Segment (2) 9 pts
No formula derivation (−3)

Circular segment 9 pts
No formula derivation (−3)

direction 3 pts

* Every calculation should start from the Biot-Savart Law.

Circular Segment



$$d\vec{B}_c = \frac{\mu_0}{4\pi}\frac{I\,d\vec{s}\times\hat{r}}{r^2}$$

$$= \frac{\mu_0}{4\pi}\frac{I\,ds\sin 90^\circ}{R^2}$$

$$\therefore |\vec{B}_c| = \frac{\mu_0}{4\pi}\frac{I}{R^2}\oint ds = \frac{\mu_0}{4\pi}\frac{I}{R^2}2\pi R = \frac{\mu_0 I}{2R}$$

$$\therefore \vec{B}_c = \frac{\mu_0 I}{2R}(-\hat{z})$$

Total $\vec{B}$ at $\vec{P}$

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_c$$

$$= 0 + \frac{\mu_0 I}{4\pi R}\hat{z} - \frac{\mu_0 I}{2R}\hat{z}$$

$$= \frac{\mu_0 I}{2R}\left(\frac{1}{2\pi} - 1\right)\hat{z}$$

$$= -4.40\times10^{-5}\,\hat{z}\ \mathrm{T}$$