Consider modeling a bolt of lightning as a long cylinder with a current density given by \( j = j_0(1 - \alpha R^3) \), where \( \alpha \) is a constant and \( j = 0 \) at the outside radius \( R_0 \) where \( R_0 = 4.50 \text{ cm} \). The total current is 65,500 Amperes (a typical number).

(a) Calculate \( \alpha \) (numerical value with units).

\[
\begin{align*}
\text{Let} & \quad j(R_0) = j_0 (1 - \alpha R_0^3) = 0 = \quad \Rightarrow \quad \alpha = 1/R_0^3 = \frac{1}{(4.5 \times 10^{-2})^3} = 1.1 \times 10^4 \quad \text{m}^{-3}
\end{align*}
\]

(b) \[
I = \int_0^{R_0} j r dr = 2\pi j_0 \int_0^{R_0} r(1 - \alpha r^3) dr = 2\pi j_0 \int_0^{R_0} \frac{R_0^2 - \alpha \frac{R_0^5}{5}}{2} dr 
\]
\[
\Rightarrow \quad j_0 = \frac{5I}{3\pi R_0^2} = \frac{5 \times 65,500}{3\pi(4.5 \times 10^{-2})^2} = 1.72 \times 10^7 \quad \text{A/m}^2
\]

(c) \[
2\pi R B = \mu_0 I_{\text{ins}}
\]
\[
I_{\text{ins}} = \int_0^{R_0} j_0 (1 - \alpha r^3) 2\pi r dr = 2\pi j_0 \int_0^{R_0} \frac{R_0^2}{2} - \alpha \frac{R_0^5}{5} dr = \]
\[
\Rightarrow \quad B = \mu_0 j_0 \int_0^{R_0} \left( \frac{R_0^2}{2} - \alpha \frac{R_0^5}{5} \right) dr = \mu_0 j_0 \int_0^{R_0} \left( \frac{1}{2} - \frac{1}{5} \left( \frac{R}{R_0} \right)^3 \right) 
\]
\[
\text{for } R = 2.5 \times 10^{-2} \text{ m,} \quad B = 4\pi \times 10^{-7} \times 1.72 \times 10^7 \times 2.5 \times 10^{-2} \times \left[ 1/2 - 1/5 \left( \frac{2.5}{4.5} \right)^3 \right] = 0.252 \quad \text{T}
\]