

FINAL EXAM

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Discussion Section # _____

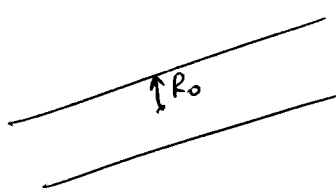
SHOW ALL WORK!!!!

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!

Use the conversion constants and data given on the front page.

Consider modeling a bolt of lightning as a long cylinder with a current density given by $j = j_0(1 - \alpha R^3)$, where α is a constant and $j = 0$ at the outside radius R_0 where $R_0 = 4.50$ cm. The total current is 65,500 Amperes (a typical number).

- 15 (a) Calculate α (numerical value with units).
 10 (b) Find j_0 (numerical value with units).
 5 (c) What is the magnetic field a distance $R = 2.50$ cm from the center of the current distribution? [If you cannot do (a) and (b) do this symbolically.]



$$(a) \quad j(R_0) = j_0(1 - \alpha R_0^3) = 0 \Rightarrow$$

$$\Rightarrow \alpha = 1/R_0^3 = \frac{1}{(4.5 \cdot 10^{-2})^3} = \boxed{1.10 \cdot 10^{+4} \text{ (m}^{-3}\text{)}}$$

$$(b) \quad I = \int_0^{R_0} j \cdot 2\pi r dr = 2\pi j_0 \int_0^{R_0} r(1 - \alpha r^3) dr =$$

$$= 2\pi j_0 \left[\frac{R_0^2}{2} - \alpha \frac{R_0^5}{5} \right] \quad \text{substituting } \alpha \text{ from (a) we get}$$

$$I = 2\pi j_0 \left[\frac{R_0^2}{2} - \frac{R_0^5}{5R_0^3} \right] = \pi j_0 \left[R_0^2 - \frac{2}{5} R_0^2 \right] = \frac{3\pi j_0 R_0^2}{5}$$

$$\Rightarrow j_0 = \frac{5I}{3\pi R_0^2} = \frac{5 \cdot 65,500}{3\pi (4.5 \cdot 10^{-2})^2} = \boxed{1.72 \cdot 10^7 \text{ (A/m}^2\text{)}}$$

$$(c) \quad 2\pi R B = \mu_0 I_{ins}$$

$$I_{ins} = \int_0^R j_0(1 - \alpha r^3) 2\pi r dr = 2\pi j_0 \left[\frac{R^2}{2} - \alpha \frac{R^5}{5} \right] \Rightarrow$$

$$\Rightarrow B = \mu_0 j_0 \left[\frac{R}{2} - \alpha \frac{R^4}{5} \right] = \mu_0 j_0 \left[\frac{R}{2} - \frac{R^4}{5R_0^3} \right] =$$

$$= \mu_0 j_0 R \left[\frac{1}{2} - \frac{1}{5} \left(\frac{R}{R_0} \right)^3 \right]$$

for $R = 2.5 \cdot 10^{-2} \text{ m}$

$$B = \frac{4\pi \cdot 10^{-7} \cdot 1.72 \cdot 10^7 \cdot 2.5 \cdot 10^{-2} \cdot \left[\frac{1}{2} - \frac{1}{5} \left(\frac{2.5}{4.5} \right)^3 \right]}{1} = \boxed{0.252 \text{ T}}$$