A long, straight wire carries a current given by $I = I_0 e^{-kt}$, where $I_0$ and $k$ are constant.

Nearby is a rectangular loop of wire with the dimensions shown. Both the straight wire and the rectangular loop are in the plane of the paper.

(a) Calculate an expression for the flux through the rectangular loop as a function of time.

(b) Calculate the current through the resistor as a function of time.

The B field due to a long straight wire is, by Ampère,

$$B = \frac{\mu_0 I}{2\pi r}$$

\[ \text{(By definition, flux } \Phi = \int B \cdot dA \text{ in this case)} \]

$$\Phi = \int_B \left( \frac{\mu_0 I}{2\pi r} \right) (L \, dr) = \frac{\mu_0 I L}{2\pi} \ln \left( \frac{b}{a} \right)$$

$$\Phi = \frac{\mu_0 I_0 L \ln \left( \frac{b}{a} \right) \cdot e^{-kt}}{2\pi}$$

\[ \text{(By Ohm's law, } I_R = \frac{E_{in}}{R} \text{ where } E_{in} \text{ is induced emf).} \]

\[ \text{(By Faraday's law, } E_{in} = -\frac{d\Phi}{dt} = -\frac{\mu_0 I_0 L \ln \left( \frac{b}{a} \right) \cdot (-k) e^{-kt}}{2\pi} \text{)} \]

$$I_R = \frac{\mu_0 I_0 L K E_{in} \cdot e^{-kt}}{2\pi R} \text{ clockwise}$$