A long cylindrical conductor of radius $R$ carries current $I$ of uniform density $J = I/\pi R^2$. Find the magnetic flux per unit length through the area indicated in the drawing.

1) **First calculate $B$ at distance $r < R$ from axis:**

$$\oint B \cdot d\ell = \mu_0 i_{\text{enclosed}} \quad \Rightarrow B = \frac{\mu_0 J}{2} r$$

So,

$$d\Phi = B(r) dr L$$

$$\Phi = \int_0^R \frac{\mu_0 J r L}{2} dr = \frac{\mu_0 J L R^2}{2}$$

**But $\pi R^2 J = I$ so**

$$\Phi = \frac{L \mu_0 J R^2 \pi}{4\pi} = \frac{L \mu_0 J}{4\pi}$$

This is flux for length $L \rightarrow$ so

$$\frac{\Phi}{L} = \text{Flux per unit length} = \frac{\mu_0 J}{4\pi}$$