Physics 2220
Spring 2006
George A. Williams

FOURTH MIDTERM

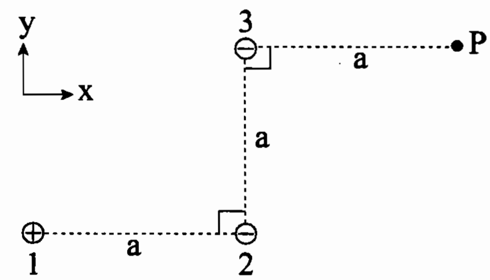
3

Name: *Solution* Student ID #: ____________

Discussion Instructor (circle): Barcikowski El-Gendy Johnson Rodriguez

<u>SHOW ALL WORK!!!!!</u>
<u>REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!</u>
<u>Use the conversion constants and data given on the front page.</u>

Three long straight wires are carrying current perpendicular to the paper. ⊕ means current out of the page, ⊖ means current into the page. Using the current values given, calculate the magnetic field at point P.

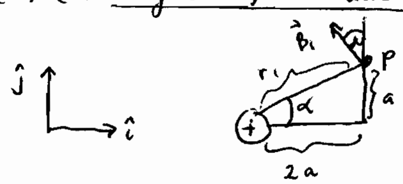


15 pts. (a) Express the magnetic field in î, ĵ notation.
10 pts. (b) Calculate the magnitude and direction of the field. Express the direction as an angle measured counter clockwise from the positive x-direction. Show on a drawing how you define this angle.

a = 4.50 cm; $I_1 = +10.25$ A; $I_2 = -5.75$ A; $I_3 = 2.30$ A

From the Biot-Savart law, the magnetic field at point P due to a long, straight wire is $\vec{B}(P) = \frac{\mu_0 I}{2\pi r}\hat{r}^{\perp}$, where $\hat{r}^{\perp}$ is a unit vector defined by the right hand thumb rule, with the the thumb in the direction of the current, and the fingers closed in the direction of $\hat{r}^{\perp}$:

(a) (i) <u>Magnetic field due to wire 1, $\vec{B}_1$:</u>



$$r_1 = \sqrt{(2a)^2 + a^2} = \sqrt{5}\,a$$

$$\sin\alpha = \frac{a}{r_1} = \frac{1}{\sqrt{5}}$$

$$\cos\alpha = \frac{2a}{r_1} = \frac{2}{\sqrt{5}}$$

$$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi r_1}(-\sin\alpha\,\hat{i} + \cos\alpha\,\hat{j}) = \frac{\mu_0 I_1}{10\pi a}(-\hat{i} + 2\hat{j}) = (-9.11\times10^{-6}\,\hat{i} + 1.822\times10^{-5}\,\hat{j})\,T$$

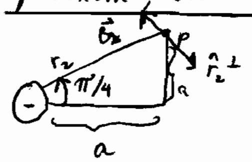
$$|\vec{B}_1| = 2.037\times10^{-5}\ T.$$

$(\mu_0 = 4\pi\times10^{-7}\ N/A^2)$

(ii) <u>Magnetic field due to wire 2, $\vec{B}_2$:</u>
Since $I_2 < 0$, the current is flowing <u>out</u> of the page, so $\vec{B}_2$ and $\hat{r}_2^{\perp}$ have opposite directions

$\longrightarrow$

problem 3 continued



$$r_2 = \sqrt{a^2 + a^2} = \sqrt{2}\, a$$

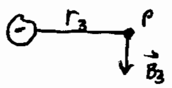
I_2 is negative

$$\vec{B}_2 = \frac{\mu_0 I_2}{2\pi r_2} \hat{r}_2^{\perp} = \frac{\mu_0 I_2}{2\pi a \sqrt{2}} \left(\frac{\sqrt{2}}{2} \hat{\imath} - \frac{\sqrt{2}}{2} \hat{\jmath} \right) = \frac{\mu_0 I_2}{4\pi a} (\hat{\imath} - \hat{\jmath}) \Rightarrow$$

$$\vec{B}_2 = \left(-1.27 \times 10^{-5}\, \hat{\imath} + 1.27 \times 10^{-5}\, \hat{\jmath}\right) T.$$

$$|\vec{B}_2| = 1.807 \times 10^{-5}\, T.$$

(iii) Magnetic Field due to wire 3, $\vec{B}_3$:



$$r_3 = a$$

$$\vec{B}_3 = -\frac{\mu_0 I_3}{2\pi a} \hat{\jmath} = -1.022 \times 10^{-5}\, \hat{\jmath} \quad T$$

$$|\vec{B}_3| = 1.022 \times 10^{-5}\, T.$$

Total Magnetic Field: $\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$:

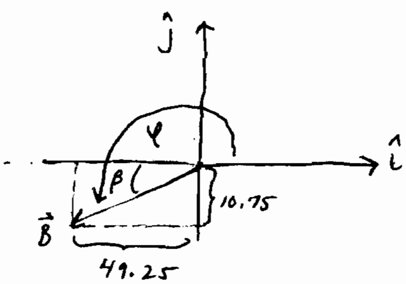
$$\vec{B} = \frac{\mu_0}{20\pi a} \left\{ 2 I_1 (-\hat{\imath} + 2\hat{\jmath}) + 5 I_2 (\hat{\imath} - \hat{\jmath}) - 10 I_3 \hat{\jmath} \right\}$$

$$= \frac{\mu_0}{20\pi a} \left\{ (-20.5 - 28.75)\hat{\imath} + (41 - 28.75 - 23)\hat{\jmath} \right\} = \frac{\mu_0}{20\pi a} (-49.25\, \hat{\imath} - 10.75\, \hat{\jmath})$$

$$= \frac{10^{-7}}{0.225} (-49.25\, \hat{\imath} - 10.75\, \hat{\jmath}) = \boxed{\left(-2.189 \times 10^{-5}\, \hat{\imath} - 4.778 \times 10^{-6}\, \hat{\jmath}\right) T = \vec{B}_{Total}}$$

(b) Magnitude $= |\vec{B}| = \frac{\mu_0}{20\pi a} \sqrt{\underbrace{(10.75)^2}_{B_y^2} + \underbrace{(49.25)^2}_{B_x^2}} = \frac{4\pi \times 10^{-7}}{20\pi (0.045)} \sqrt{(10.75)^2 + (49.25)^2}$

$$\Rightarrow \boxed{|\vec{B}| = 2.262 \times 10^{-7}\, T}$$



$$\tan\beta = \frac{B_y}{B_x}$$

$$\varphi = \pi + \beta$$

$$10.75 = 49.25 \tan\beta \Rightarrow \boxed{\varphi = 3.356\ \text{rad} = 192.313^\circ}$$