7. Given a long, straight wire carrying current to the right as shown. A rectangular loop of wire is placed near the wire as shown. If the current in the wire is given by \( I = I_0 \sin \omega t \), find an expression for the voltage across the resistor \( R \) as a function of time.

\[
B(r,t) = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I_0}{2\pi r} \sin \omega t \quad (5 \text{ points})
\]

The magnetic flux through the rectangular loop is:

\[
\Phi_R(t) = \oint B \cdot d\mathbf{s} = \int B \cdot ds
\]

\[
= \int_{2a}^{3a} (\frac{\mu_0 I_0}{2\pi r}) (Ldr) \quad (ds = Ldr)
\]

\[
= \frac{\mu_0 I_0 \omega \sin \omega t}{2\pi} L \left[ \ln \frac{3a}{2a} \right]
\]

\[
= \frac{\mu_0 I_0 L \ln(\frac{3}{2})}{2\pi} \sin \omega t \quad (13 \text{ points})
\]

\[
V_R(t) = \varepsilon(t) = -\frac{d\Phi_R}{dt} = -\frac{\mu_0 I_0 L \ln(\frac{3}{2})}{2\pi} \omega \cos \omega t
\]

(7 points)