

FIFTH MIDTERM

3

Name: _____

Discussion Instructor (circle): Billeter Blake Herring Young

Discussion Section # _____

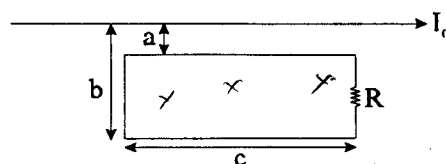
Student ID #: _____

SHOW ALL WORK!!!!

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!

Use the conversion constants and data given on the front page.

A rectangular conducting wire has the shape and dimensions as shown. The upper wire and the rectangle are in the plane of the paper.



- (a) [8 pts.] If the current in the upper wire has a steady value I_0 , calculate the magnetic flux through the rectangular loop.
- (b) [10 pts.] If the current in the upper wire is given by $I = I_0 \sin \omega t$, calculate the current in the rectangular loop as a function of time.
- (c) [7 pts.] If the positive direction of the upper current is to the right, as shown, calculate the direction of the current in (b) at $t = 0$, clockwise or counterclockwise. For full credit, state clearly your reasoning.

$$a) \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}} \Rightarrow B = \frac{\mu_0 I_0}{2\pi r}$$

$$\Phi = \int \vec{B} \cdot d\vec{A} = \int_a^b \frac{\mu_0 I}{2\pi r} c dr = \frac{c\mu_0 I_0}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$b) \mathcal{E} = -\frac{d\Phi}{dt} = -\frac{c\mu_0}{2\pi} \ln\left(\frac{b}{a}\right) \frac{d}{dt}(I_0 \sin \omega t) = -\frac{c\mu_0 I_0 \omega}{2\pi} \ln\left(\frac{b}{a}\right) \cos(\omega t)$$

$$I = \frac{\mathcal{E}}{R} = -\frac{c\mu_0 I_0 \omega}{2\pi R} \ln\left(\frac{b}{a}\right) \cos(\omega t)$$

c) Counterclockwise

Since just after $t=0$, the current $I = I_0 \sin \omega t$ start to increase, the magnetic field created by wire, which is into paper start to increase too, so the magnetic flux ^{in loop} produced by wire start to increase too. According to Lenz's law, the magnetic field created by the induced current opposes the change in magnetic flux, so the induced magnetic field will be out of paper, so we can get the induced current ^{is} counterclockwise (RH Rule) (in loop)