

FINAL EXAM

Name (print) Godfrey-Smith Name (signed) \_\_\_\_\_

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Discussion Section # \_\_\_\_\_

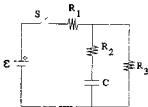
**SHOW ALL WORK!!!**

**REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!**

Use the conversion constants and data given on the front page.

Given the circuit shown.

- (a) Calculate the charge on the capacitor 1.50 time constants after the switch is closed. (Numerical value.)  
 (b) Find the current in  $R_3$  at  $t = \infty$ . (Numerical value.)  
 (c) Calculate the charge on the capacitor 0.350 s after the switch is opened, after being closed for a long time.  
 (d) Calculate, in complete detail, the time constant for charging the capacitor. No short cuts from other classes allowed. (Numerical value.)



$\mathcal{E} = 90.0 \text{ V}; R_1 = 150 \Omega; R_2 = 200 \Omega; R_3 = 300 \Omega; C = 450 \mu\text{F}$

6 A)  $Q(t) = Q_{\infty} (1 - e^{-t/\tau})$  for charging

$Q_{\infty} = C V_{R_3} = C \frac{\mathcal{E}}{R_1 + R_3} \cdot R_3 = 2.7 \times 10^{-2} \text{ C}$

$Q(1.5\tau) = 2.7 \times 10^{-2} \text{ C} (1 - e^{-1.5}) = 2.10 \times 10^{-2} \text{ C}$

6 B) at  $t \rightarrow \infty$   $I_2 = 0$ ,  $I_1 = I_3 = \frac{\mathcal{E}}{R_1 + R_3} = \frac{90}{150 + 300} = 0.20 \text{ A}$

6 C)  $Q_{\infty} = C V_{R_3} = 2.70 \times 10^{-2} \text{ C}$  as in part A.

After switch is open,  $R_{\text{eq}} = R_2 + R_3 = 500 \Omega$ ,  $\tau = R_{\text{eq}} C = 0.225 \text{ s}$

$Q(t) = Q_{\infty} e^{-t/\tau} = 2.7 \times 10^{-2} e^{-.350/0.225} = 5.70 \times 10^{-3} \text{ C}$

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[Cont.]

D) 1 junction, 2 loops:

$$\textcircled{1} I_1 = I_2 + I_3$$

$$7 \textcircled{2} \mathcal{E} - R_1 I_1 - R_2 I_2 - \frac{Q}{C} = 0$$

$$\textcircled{4} I_2 = \frac{dQ}{dt}$$

$$\textcircled{3} \frac{Q}{C} + R_2 I_2 - R_3 I_3 = 0$$

Solve for  $Q + \frac{dQ}{dt}$ :

$$\textcircled{5} I_3 = \frac{Q}{R_3 C} + \frac{R_2}{R_3} I_2, \text{ from } \textcircled{3}$$

$$5 \textcircled{6} \mathcal{E} = (R_1 + R_2) I_2 + \frac{R_1 Q}{R_3 C} + \frac{R_1 R_2}{R_3} I_2 + \frac{Q}{C}, \text{ from } \textcircled{1} + \textcircled{2}$$

$$\textcircled{7} \mathcal{E} = \frac{(R_1 + R_2) R_3 + R_1 R_2}{R_3} I_2 + \frac{R_1 + R_3}{R_3} \frac{Q}{C}, \text{ rearranging terms}$$

$$\textcircled{8} \frac{R_3 \mathcal{E}}{(R_1 + R_2) R_3 + R_1 R_2} = \frac{dQ}{dt} + \underbrace{\frac{R_1 + R_3}{(R_1 + R_2) R_3 + R_1 R_2} \cdot \frac{Q}{C}}_{= \tau^{-1}}, \text{ from } \textcircled{7} + \textcircled{4}$$

$$\text{So } \tau = \left[ \frac{(R_1 + R_2) R_3 + R_1 R_2}{R_1 + R_3} \right] C = \frac{350 \cdot 300 + 150 \cdot 200}{(150 + 300)} \cdot 480 \cdot 110^{-6} \text{ s}$$

$$\tau = 300 \Omega \times 450 \mu\text{F} = 0.135 \text{ s}$$