Physics 302
Spring Quarter 1990
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George Williams

Avg. 19.94

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FINAL EXAM

Name (print)_____ Name (signed)_____

Discussion Instructor (circle one): Baselgia Morrill Reeve Stoops Zhang

Discussion Section #_____

SHOW ALL WORK!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

Consider a spherical uniform charge distribution of radius R_0. The charge density is ρ.

(30)

(a) Find the value of r such that half the total charge is inside r and call this value r_1.
(b) Find the electric field at $r = r_1$.
(c) Calculate the energy stored between $r = \frac{1}{2} r_1$ and $r = r_1$.

Solution:

(10) (a) Assume the total charge is Q, the charge density is ρ

It is clear $Q = \frac{4\pi}{3}\rho R_0^3$

Since $\frac{4\pi}{3}\rho r_1^2 = \frac{Q}{2}$ $\rightarrow$ $\boxed{r_1 = \frac{R_0}{\sqrt[3]{2}}}$

(10) (b) Using Gauss's law:

$$\int \vec{E}\cdot d\vec{S} = \frac{Q(r)}{\varepsilon_0}$$

$$4\pi r_1^2 E = \frac{2\pi\rho R_0^3}{3\varepsilon_0} \rightarrow \boxed{E = \frac{\pi\rho}{6\varepsilon_0}\frac{R_0^3}{r_1^2} = \frac{\sqrt[3]{4}\rho R_0}{6\varepsilon}}$$

(10) (c) We know the energy density is: $u = \frac{1}{2}\varepsilon_0 E^2$

Therefore, the energy stored between $r = \frac{1}{2}r_1$ and $r = r_1$ is:

$$U = \int_{\frac{r_1}{2}}^{r_1} u\, dV = \int_{\frac{r_1}{2}}^{r_1} \frac{1}{2}\varepsilon_0 E^2 \cdot 4\pi r^2 dr$$

Since $E \cdot 4\pi r^2 = \frac{1}{\varepsilon_0}\frac{4}{3}\pi r^3 \rho \rightarrow E = \frac{\rho r}{3\varepsilon_0}$

$$U = \frac{1}{2}\varepsilon_0 \int_{\frac{r_1}{2}}^{r_1} \left(\frac{\rho r}{3\varepsilon_0}\right)^2 4\pi r^2 dr = \frac{2\pi\varepsilon_0\rho^2}{9\varepsilon_0^2}\int_{\frac{r_1}{2}}^{r_1} r^4 dr$$

$$= \frac{2\pi\rho^2}{9\varepsilon_0}\times\frac{1}{5}\left[r_1^5 - \left(\frac{r_1}{2}\right)^5\right] = \boxed{\frac{31\pi r_1^5\rho^2}{720\varepsilon_0} = \frac{31\pi R_0^5\rho^2}{1440\sqrt[3]{4}\varepsilon_0}}$$