

FINAL EXAM

Name (print) C. Stone Name (signed) _____

Discussion Instructor (circle one): Cady McAllister Molina Stone

Discussion Section #: _____

Note: Both λ and Q are positive quantities.

SHOW ALL WORK!!!!

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!

Use the conversion constants and data given on the front page.

Consider an infinite line of charge, with charge density given by $-\lambda \hat{x}/m$. Calculate the work needed to move a charge $+Q$ from point B to point A. Give both the magnitude and the sign.

First, use Gauss's law to find the magnitude of the electric field at a distance r from a line of (negative) charge:

$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$, or [using a cylindrical Gaussian surface of length l and radius r]

$$2\pi l r E = \left| \frac{-\lambda l}{\epsilon_0} \right| \Rightarrow E = \frac{\lambda}{2\pi\epsilon_0 r} \text{ or } \frac{2k\lambda}{r}$$



As a vector, $\vec{E} = \frac{-\lambda \hat{r}}{2\pi\epsilon_0 r} = -\frac{2k\lambda}{r} \hat{r}$ [points radially inwards towards the wire]

Now calculate $V_A - V_B = - \int_B^A \vec{E} \cdot d\vec{s} = - \int_{r=b}^{r=a} \left(-\frac{2k\lambda \hat{r}}{r} \right) \cdot (\hat{r} dr)$

$= 2k\lambda \int_b^a \frac{dr}{r} = 2k\lambda \ln\left(\frac{a}{b}\right)$. Since $a < b$, this potential difference

is negative, as expected, given the direction of \vec{E} . (\vec{E} always points from regions of higher potential towards regions of lower potential.)

The work W that would have to be done by an external force to move a charge Q from B to A at constant velocity (i.e., with no acceleration) is then

$W = Q(V_A - V_B) = \boxed{2kQ\lambda \ln\left(\frac{a}{b}\right)}$. The sign of the

work is negative.