Physics 172
Winter Quarter 1988
March 15, 1988
George Williams

Students : 264
Average : 25.2 (± 5.4)
Total : 30.

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FINAL EXAM

Name (print) Rong-Zhen Qian. Name (signed)__________

Discussion Instructor (circle one): Cady McAllister Molina Stone

Discussion Section #: 30 TOTAL

SHOW ALL WORK!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

Consider a positively charged sphere whose charge density can be expressed as $\rho = \rho_o(1 - \alpha R)$, where the radius of the sphere is R_o. If the charge density goes to zero at $R = R_o$, and Q is the total charge, calculate:

(a) the value of α (in terms of R_o);
(b) the value of ρ_o in terms of Q, R_o and numbers only;
(c) the electric field at $R = R_o/3$ (eliminate ρ_o and α from your final expression).

5 (a). $\rho(R) = \rho_o(1-\alpha R)$.

$\rho(R_o) = \rho_o(1-\alpha R_o) = 0 \quad \therefore \quad 1-\alpha R_o = 0, \Rightarrow \boxed{\alpha = 1/R_o}$

10 (b). Total charge $Q = \int_{\text{sphere}} \rho\, dv = \int_0^{R_o} \rho_o(1-\alpha R)\cdot 4\pi R^2 dR = 4\pi\rho_o \int_0^{R_o}(R^2 - \alpha R^3)\, dR$

$= 4\pi\rho_o\,(R_o^3/3 - \alpha R_o^4/4) \overset{\alpha = 1/R_o}{=\!=} 4\pi\rho_o\,(R_o^3/3 - R_o^3/4) = \frac{\pi\rho_o}{3} R_o^3$

$\Rightarrow \rho_o = \boxed{\frac{3Q}{\pi R_o^3}}$

15 (c). By Gauss' law

at R: $E \cdot 4\pi R^2 = \frac{1}{\epsilon_o} Q(R)$; $Q(R)$ ~ total charge inside the sphere of radius R.

$= \frac{1}{\epsilon_o}\int_0^R \rho_o(1-\alpha R)\cdot 4\pi R^2 dR$

$= \frac{4\pi\rho_o}{\epsilon_o}\,(R^3/3 - \alpha R^4/4)$. $\quad \because \begin{cases}\alpha = 1/R_o \\ \rho_o = \frac{3Q}{\pi R_o^3} \\ R = R_o/3\end{cases}$

$= \frac{4\pi}{\epsilon_o}\cdot\frac{3Q}{\pi R_o^3}\left[\frac{R_o^3}{81} - \frac{R_o^3}{324}\right]$

$= \frac{Q}{9\epsilon_o}$

$\therefore E = \frac{1}{4\pi R^2}\cdot\frac{Q}{9\epsilon_o} = \frac{1}{4\pi\cdot\frac{1}{9}R_o^2}\cdot\frac{Q}{9\epsilon_o} = \boxed{\frac{Q}{4\pi\epsilon_o R_o^2}}$