Physics 2220
Spring 2000
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FINAL EXAM

7

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Discussion Section # _____

SHOW ALL WORK!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

A long non-conducting rod of radius R_o is electrically charged. The charge density within the rod is uniform and can be represented as $+\lambda$(C/m) along the rod. (A length ℓ of the rod has a total charge $\lambda\ell$.)

(a) Find an expression for the electric field for any value of $R > R_o$.
(b) Find an expression for the electric field for any value of $R < R_o$.
(c) What is the magnitude of the potential difference between $R_o/2$ and R_o.
(d) Determine the energy stored in the electric field, per meter length, between R_o and $2R_o$.

a) $R > R_o \quad \oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$

$\oint \vec{E}\cdot d\vec{A} = E\cdot\oint dA = 2\pi R\ell E$

$q_{enc} = \lambda\ell \Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 R} \quad \left(E = \frac{2k\lambda}{R}\right)$

b) $R < R_o$

$q_{enc} = \lambda\ell\cdot\frac{R^2}{R_o^2} \Rightarrow E = \frac{\lambda R}{2\pi\varepsilon_0 R_o^2}$

c) $\Delta V = -\int_{R_o/2}^{R_o} \vec{E}\cdot d\vec{\ell} \quad \vec{E}$ for $R < R_o$

$\Delta V = -\frac{\lambda}{2\pi\varepsilon_0 R_o^2}\int_{R_o/2}^{R_o} R\,dR = -\frac{3\lambda}{16\pi\varepsilon_0}$

d) $U_e = \frac{1}{2}\varepsilon_0\int E^2\,dV \quad E$ for $R > R_o$

$\frac{U_e}{\ell} = \int_{R_o}^{2R_o}\left(\frac{1}{2}\varepsilon_0 E^2\right)2\pi R\,dR = \frac{1}{2}2\pi\varepsilon_0\int_{R_o}^{2R_o}\left(\frac{\lambda}{R\pi\varepsilon_0}\right)^2\frac{1}{R^2}R\,dR =$

$= \frac{\lambda^2}{4\pi\varepsilon_0}\int_{R_o}^{2R_o}\frac{1}{R}\,dR = \frac{\lambda^2}{4\pi\varepsilon_0}\ln 2$