

SIXTH MIDTERM

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SHOW ALL WORK!!!!

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!

Use the conversion constants and data given on the front page.

A long cylinder shaped charge distribution of radius R_0 has a charge density given by $\rho = BR^3$ (not physically sensible, but it keeps the math simple). B is a constant.

- 10 (a) Find the total charge for a length l of this cylinder.
 10 (b) Calculate the electric field at an arbitrary value of R where $R < R_0$.
 10 (c) Calculate the magnitude of the electric potential difference between $R = 0$ and $R = \frac{2}{3}R_0$.

$$\begin{aligned}
 a) \quad Q &= \int \rho dV = \int \rho \cdot R dR d\theta dz \\
 &= \int_0^{2\pi} d\theta \int_0^{R_0} BR^3 \cdot R dR \int_0^l dz \\
 &= 2\pi l B \int_0^{R_0} R^4 dR = \frac{2\pi l B}{5} R_0^5.
 \end{aligned}$$

$$b) \quad \oint \vec{E} \cdot d\vec{s} = \frac{1}{\epsilon_0} \Sigma Q$$

At arbitrary R , the enclosed charge is $\frac{2\pi l B}{5} R^5$.

$$\text{and } \oint \vec{E} \cdot d\vec{s} = E \cdot 2\pi R \cdot l = \frac{2\pi l B}{5} R^5.$$

$$\Rightarrow E = \frac{B}{5\epsilon_0} R^4.$$

$$c) \quad U = -\int \vec{E} \cdot d\vec{r} = -\int_0^{\frac{2}{3}R_0} \frac{B}{5\epsilon_0} R^4 dR.$$

$$= -\frac{B}{25\epsilon_0} R^5 \Big|_0^{\frac{2}{3}R_0} = -B \frac{1}{25\epsilon_0} \left(\frac{2}{3}\right)^5 R_0^5.$$