

SIXTH MIDTERM

6

Name: _____

Jiansheng Li

Discussion Instructor (circle): Billeter Diwekar Kennedy Murray Whitaker

Discussion Section # _____

Student ID #: _____

SHOW ALL WORK!!!!

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!

Use the conversion constants and data given on the front page.

A bolt of lightning has a current density distribution given by $j = AR^4$ for $R < R_0$ and 0 for $R > R_0$ (not physical, but keeps the math simple).

- (a) Calculate the magnetic field at a radius R , where $R < R_0$.
(b) Find the energy in the magnetic field for a length l and between $R = 0$ and $R = \frac{3}{4}R_0$.

15(a) $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclose}}$

$$2\pi R B = \int_0^R AR^4 dS = \int_0^R AR^4 2\pi R dR = 2\pi A \frac{R^6}{6}$$
$$\therefore B = \frac{\mu_0 AR^5}{6}$$

15(b) energy density $\frac{B^2}{2\mu_0} = \frac{\mu_0 A^2 R^{10}}{72}$

$$E = \int \frac{B^2}{2\mu_0} dV = \int_0^{\frac{3R_0}{4}} \frac{\mu_0 A^2 R^{10}}{72} 2\pi R l dR$$
$$= \frac{\pi \mu_0 A^2 l}{36} \int_0^{\frac{3R_0}{4}} R^{11} dR = \frac{\pi \mu_0 A^2 l}{432} \left(\frac{3}{4}R_0\right)^{12}$$
$$= 7.33 \times 10^{-5} \pi A^2 \mu_0 R_0^{12}$$