

FINAL EXAM

6

Name: YONG HAN

Discussion Instructor (circle): Andrade El-Gendy Mimoto Owen

Discussion Section # _____

Student ID #: _____

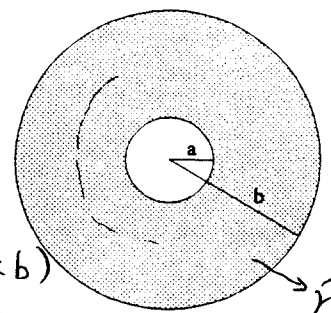
SHOW ALL WORK!!!!

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!

Use the conversion constants and data given on the front page.

A long cylindrical non-conductor has the cross-section shown. Charge is confined to the shaded region and is governed by the charge density given by $\rho(R) = \rho_0 R^3$. $\rho(R) = 0$ for $R > b$ or $R < a$. The charge is positive.

- (a) Calculate the total charge in a length l of this cylinder.
- (b) Calculate the electric field at a point R where $R < a$.
- (c) Calculate the electric field at a point R where $R > b$.
- (d) Calculate the electric field at a point R where $a < R < b$.



$$V = \pi R^2 l, \quad dV = 2\pi R l dR, \quad \rho(R) = \begin{cases} 0 & (R < a) \\ \rho_0 R^3 & (a < R < b) \\ 0 & (R > b) \end{cases}$$

6 (a) $Q = \int_a^b \rho(R) dV = \int_a^b \rho_0 R^3 \cdot 2\pi R l dR$
 $= 2\pi \rho_0 l \int_a^b R^4 dR = \frac{2}{5} \pi \rho_0 l R^5 \Big|_a^b = \boxed{\frac{2}{5} \pi \rho_0 l (b^5 - a^5)}$

6 (b) $\int_S \vec{E} \cdot d\vec{S} = \int_0^R \rho(R) dV = 0, \quad \boxed{\vec{E} = 0} \quad (R < a)$

6 (c) $\int_S \vec{E} \cdot d\vec{S} = \frac{Q}{\epsilon_0}, \quad E \cdot 2\pi R l = \frac{1}{\epsilon_0} \frac{2}{5} \pi \rho_0 l (b^5 - a^5),$
 $\vec{E} = \boxed{\frac{\rho_0}{5\epsilon_0} \frac{b^5 - a^5}{R} \hat{r}}, \quad (\hat{r} \text{ is the unit vector.})$

12 (d) $\int_S \vec{E} \cdot d\vec{S} = \frac{1}{\epsilon_0} \int_a^R \rho(R) dV,$
 $E \cdot 2\pi R l = \frac{1}{\epsilon_0} \cdot \frac{2}{5} \pi \rho_0 l (R^5 - a^5),$
 $\vec{E} = \boxed{\frac{\rho_0}{5\epsilon_0} \frac{R^5 - a^5}{R} \hat{r}}$