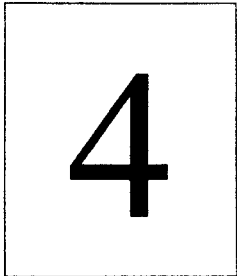


# FINAL EXAM



Bogdan Popescu

Name: \_\_\_\_\_

Discussion Instructor (circle): Billeter      Blake      Herring      Young

Discussion Section # \_\_\_\_\_

Student ID #: \_\_\_\_\_

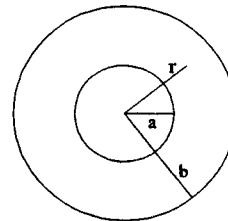
**SHOW ALL WORK!!!!**

**REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!**

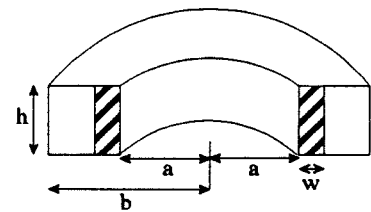
**Use the conversion constants and data given on the front page.**

A toroid with rectangular cross section as shown has 1325 evenly spaced turns of wire. The wire carries a current of 1.75 A.

- (a) [10 pts.] Calculate the magnetic field inside the toroid at an arbitrary value of  $r$  for  $a < r < b$ .  
 (b) [10 pts.] Calculate the self-inductance of the toroid.  
 (c) [10 pts.] Calculate the magnetic energy stored between  $r = 3.00$  cm and  $r = 3.50$  cm in the toroid. [This is shown as the cross-hatched region.]



Top View



Cross Section

$a = 3.00$  cm;  $b = 4.25$  cm;  $h = 1.10$  cm

a)  $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$ ,  $B 2\pi r = \mu_0 N I_0$        $B = \frac{\mu_0 N I_0}{2\pi r}$  (10 points)

b)  $\Phi = \int_A \vec{B} \cdot d\vec{A}$ ,  $dA = h dr$ ,  $\Phi = \int_a^b \frac{\mu_0 N I_0}{2\pi r} h dr = \frac{\mu_0 N I_0 h}{2\pi} \int_a^b \frac{dr}{r} = \frac{\mu_0 N I_0 h}{2\pi} \ln\left(\frac{b}{a}\right)$

$L = \frac{N\Phi}{I_0} = \frac{\mu_0 N^2 h}{2\pi} \ln\left(\frac{b}{a}\right) = 1.35 \times 10^{-3} H = 1.35 mH$  (10 points)

c)  $u_B = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0} \frac{\mu_0^2 N^2 I_0^2}{4\pi^2 r^2} = \frac{\mu_0 N^2 I_0^2}{8\pi^2 r^2}$ ,  $dV = 2\pi r h dr$

$U = \int_V u_B dV = \int_{r_1}^{r_2} \frac{\mu_0 N^2 I_0^2}{8\pi^2 r^2} 2\pi r h dr = \frac{\mu_0 N^2 I_0^2 h}{4\pi} \int_{r_1}^{r_2} \frac{dr}{r}$

$U = \frac{\mu_0 N^2 I_0^2 h}{4\pi} \ln\left(\frac{r_2}{r_1}\right) = 9.12 \times 10^{-4} J$  (10 points)