Physics 2220
Spring 2003
George A. Williams

FINAL EXAM

5

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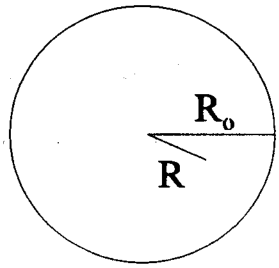
Discussion Section # ____________ Student ID #: ____________________

SHOW ALL WORK!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

A spherical distribution of negative charge on a non-conductor is described by a charge density $\rho = AR^4$, where A is a constant. The radius of the sphere is R_o



(a) *[10 pts.]* If the magnitude of the total charge on the sphere is Q_o, calculate the magnitude of A.

(b) *[8 pts.]* Calculate the magnitude of the electric field at an arbitrary interior point ($R < R_o$) of the sphere at a distance R from the center.

(c) *[7 pts.]* Calculate the magnitude of the potential difference between $R = R_o/2$ and $R = R_o$.

(d) *[5 pts.]* Determine the sign of $V(R_o) - V(R_o/2)$. For credit, you must clearly explain your reasoning.

a)

$$\int \rho \, dV = q_{enc} \quad \text{so} \quad \int_0^{R_o} AR^4 4\pi R^2 \, dR = 4\pi A \int_0^{R_o} R^6 \, dR = 4\pi A \frac{R_o^7}{7} = Q_o$$

$$\text{so} \quad A = \frac{7Q_o}{4\pi R_o^7}$$

b) Gauss's Law

$$\oint E \cdot dA = \frac{q_{enc}}{\varepsilon_o} = \frac{1}{\varepsilon_o} \int \rho \, dV$$

So

$$E \, 4\pi R^2 = \frac{1}{\varepsilon_o} \int_0^R A r^4 4\pi r^2 \, dr = \frac{4\pi}{\varepsilon_o} A \frac{R^7}{7} \quad \text{or} \quad |E| = \frac{AR^5}{7\varepsilon_o} = \frac{Q}{4\pi\varepsilon_o} \frac{R^5}{R_o^7}$$

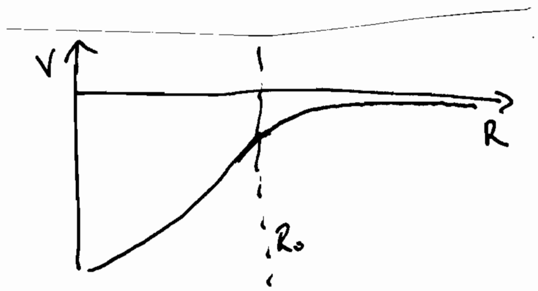
c)

$$V = -\int_{R=\frac{R_o}{2}}^{R=R_o} E \, d\ell = \frac{1}{\varepsilon_o} \int_{R=\frac{R_o}{2}}^{R=R_o} \frac{Q_o}{4\pi} \frac{R^5}{R_o^7} \, dR = \frac{Q_o}{4\pi\varepsilon_o R_o^7} \left. \frac{R^6}{6} \right|_{R=\frac{R_o}{2}}^{R=R_o} =$$

$$\frac{Q_o}{4\pi R_o^7 \varepsilon_o} \left(\frac{R_o^6}{6} - \frac{R_o^6}{6 \cdot 2^6} \right) = \frac{Q_o}{24\pi R_o \varepsilon_o} \left(1 - \frac{1}{64} \right) = \frac{Q_o}{24\pi R_o \varepsilon_o} \left(\frac{63}{64} \right)$$

d) on Back

d) A graph of the potential would look like



A quick glance shows that $V(R_0) > V(R_0/2)$ so

$V(R_0) - V(\frac{R_0}{2}) > 0$ - sign is positive.

Note that $V = -\int E \cdot dl$. In part b we calculated the magnitude of E. Because Q_0 is negative - we can write E as

$$E = -\frac{|Q_0|}{4\pi\epsilon_0} \frac{R^5}{R_0^7}$$

and get the same result.