Physics 2220
Spring 2004
George A. Williams

FINAL EXAM

6

Name: Adam Blake

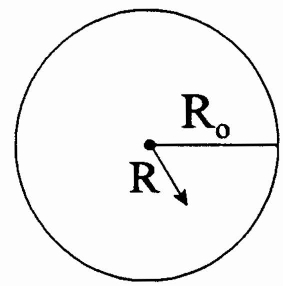
Discussion Instructor (circle): Billeter Blake Herring ~~Young~~ Gillman

Discussion Section # ____________ Student ID #: ____________________

SHOW ALL WORK!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

Given a spherically symmetric charge distribution, where the charge density is given by:

$$\rho(R) = B(1 - \alpha R^4)$$



+5 (a) At $R = R_o$ the charge density is $\rho(R_o) = 0$. Calculate α.
+6 (b) If the total charge is Q_o, calculate B.
+7 (c) Calculate the electric field at an arbitrary value of R, for $R < R_o$ (inside the sphere).
+7 (d) Calculate the magnitude of the potential difference between the center and the outer surface of the sphere.
+5 (e) If V = 0 at $R = \infty$, and the sphere is negatively charged, what is the potential at R = 0?

a) $\rho(R_o) = 0 = B(1 - \alpha R_o^4)$ so $\alpha R_o^4 = 1$ or $\boxed{\alpha = \frac{1}{R_o^4}}$

b) $Q_o = \int_0^{R_o} \rho \, dV = \int_0^{R_o} B(1 - \alpha R^4)\, 4\pi R^2 dR = 4\pi B\left(\frac{R_o^3}{3} - \alpha \frac{R_o^7}{7}\right)$

so using α from part a

$$\boxed{B = \frac{Q_o}{R_o^3} \frac{21}{16\pi}}$$

c) using Gauss' Law

$\oint E \cdot dA = \frac{1}{\epsilon_o} \int \rho \, dV$ or $E \, 4\pi R^2 = \frac{1}{\epsilon_o} 4\pi B\left(\frac{R}{3} - \alpha \frac{R^7}{7}\right)$

so $E = \frac{B}{\epsilon_o}\left(\frac{R}{3} - \alpha \frac{R^5}{7}\right)$

D) $|V| = \left|-\int E \cdot dl\right| = \int_0^{R_o} \frac{B}{\epsilon_o}\left(\frac{R}{3} - \alpha \frac{R^5}{7}\right) dR = \frac{B}{\epsilon_o}\left(\alpha \frac{R_o^6}{42} - \frac{R_o^2}{6}\right) = \frac{3 Q_o}{16\pi R_o \epsilon_o}$

e) $\Delta V_{outside} = V_\infty - V_{R_o} = \frac{Q}{4\pi \epsilon_o R}$ so $\Delta V_{outside} + \Delta V_{inside} =$

$-\frac{Q}{4\pi \epsilon_o R_o} - \frac{3 Q_o}{16\pi \epsilon_o R_o} = \boxed{\frac{-7}{16\pi \epsilon_o R_o}}$