Physics 2220
Spring 2004
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FINAL EXAM

7

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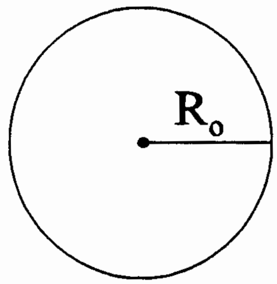
Discussion Section # ____________ Student ID #: ____________

SHOW ALL WORK!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

We model the current distribution in a cylindrical bolt of lightening where the current density is given by:

$$j(R) = j_o\,(1 - \alpha R^2)$$

where R_o = 10.5 cm.



(a) If $j(R_o) = 0$, find the value of α.
(b) If the total current is 10,000 A, calculate j_o.
(c) Calculate the magnitude of the magnetic field at an arbitrary value of R, where $R < R_o$.
(d) Calculate the magnetic energy stored in a 3.00 m length of this cylindrical bolt.

a) $j_o(1-\alpha R_o^2) = 0$

$\alpha R_o^2 = 1$

$\boxed{\alpha = \frac{1}{R_o^2} = 90.7\ \frac{1}{m^2}}$

b) $I_T = \int_0^{R_o} j_o(1-\alpha R^2)\,2\pi R\,dR$

$= 2\pi j_o\left(\frac{R_o^2}{2} - \frac{R_o^2}{4}\right)$

$= 2\pi j_o \frac{R_o^2}{4} = 10{,}000\ A$

$\boxed{j_o = \frac{20{,}000}{\pi R_o^2} = 5.77\times10^5\ \frac{A}{m^2}}$

c) $\int B \cdot dl = \mu_0 \int j(R)\, dA \qquad dA = 2\pi R\, dR$

$$2\pi R B = \mu_0 \int j_0 \left(1 - \frac{R^2}{R_0^2}\right) 2\pi R\, dR$$

$$= 2\pi \mu_0 j_0 \left[\frac{R^2}{2} - \frac{R^4}{4R_0^2}\right]$$

$$= 2\pi \mu_0 j_0 R^2 \left[\frac{1}{2} - \frac{R^2}{4R_0^2}\right]$$

$$\boxed{B = \mu_0 j_0 R \left[\frac{1}{2} - \frac{R^2}{4R_0^2}\right]}$$

d) $U_{in} = \int \frac{B^2}{2\mu_0}\, dV \qquad dV = 2\pi R l\, dR$

$$B^2 = \mu_0^2 j_0^2 R^2 \left(\frac{1}{4} - \frac{R^2}{4R_0^2} + \frac{R^4}{16R_0^4}\right)$$

$$U_{in} = \int_0^{R_0} \frac{\mu_0^2 j_0^2 R^2 \left(\frac{1}{4} - \frac{R^2}{4R_0^2} + \frac{R^4}{16R_0^4}\right)}{2\mu_0} 2\pi R l\, dR$$

$$= \pi \mu_0 j_0^2 l \left[\frac{1}{4}\int_0^{R_0} R^3\, dR - \frac{1}{4R_0^2}\int_0^{R_0} R^5\, dR + \frac{1}{16R_0^4}\int_0^{R_0} R^7\, dR\right]$$

$$= \pi \mu_0 j_0^2 l \left[\frac{R_0^4}{16} - \frac{R_0^4}{24} + \frac{R_0^4}{128}\right]$$

$$= \pi \mu_0 j_0^2 l R_0^4 \left(\frac{11}{384}\right)$$

$$= \boxed{13.7\ \text{J}}$$