FINAL EXAM

PLEASE FILL IN THE INFORMATION BELOW

Name (printed):

Name (signed):

Student ID Number

Discussion Instructor: Anthony        KC        Matt        Paul        Peter

Important!

You are not required to answer all questions in this exam. You will receive points toward your exam points for every question correctly answered. Partial credit is available, however a minimum amount of work and explanation must be shown to receive any credit (for example, a mere list of equations with no explanation of their connection to the problem will not receive credit).
Data: Use these constants (where it states, for example, 1 ft, the 1 is exact for significant figure purposes).

1 ft = 12 in (exact)  
1 m = 3.28 ft
1 mile = 5280 ft (exact)
1 hour = 3600 sec = 60 min (exact)
1 day = 24 hr (exact)
g_{\text{earth}} = 9.80 \text{ m/s}^2 = 32.2 \text{ ft/s}^2
g_{\text{moon}} = 1.67 \text{ m/s}^2 = 5.48 \text{ ft/s}^2
1 year = 365.25 days
1 kg = 0.0685 slug
1 N = 0.225 pound
1 horsepower = 550 ft\cdot\text{pounds/s} (exact)

\begin{align*}
M_{\text{earth}} &= 5.98 \times 10^{24} \text{ kg} \\
R_{\text{earth}} &= 6.38 \times 10^3 \text{ km} \\
M_{\text{sun}} &= 1.99 \times 10^{30} \text{ kg} \\
R_{\text{sun}} &= 6.96 \times 10^8 \text{ m} \\
M_{\text{moon}} &= 7.35 \times 10^{22} \text{ kg} \\
R_{\text{moon}} &= 1.74 \times 10^3 \text{ km} \\
G &= 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2 \\
k &= 8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2 \\
\varepsilon_0 &= 8.85 \times 10^{-12} \text{ C}^2/(\text{N}\cdot\text{m}^2) \\
\varepsilon_{\text{electron charge}} &= -1.60 \times 10^{-19} \text{ C} \\
m_{\text{electron}} &= 9.11 \times 10^{-31} \text{ kg} \\
\mu_o &= 4\pi \times 10^{-7} \text{ T}\cdot\text{m/A} (\text{exact}) \\
\text{N(Avogadro's No.)} &= 6.02 \times 10^{23} \text{ atoms/gm-mole} \\
&= 6.02 \times 10^{26} \text{ atoms/kg-mole} \\
1 \text{ Tesla} &= 10,000 \text{ gauss (exact)} \\
\rho(\text{H}_2\text{O}) &= 1000 \text{ kg/m}^3 \\
\cos(a \pm b) &= \cos a \cos b \mp a \sin b \\
\sin(a \pm b) &= \sin a \cos b \pm \sin b \\
m_{\text{proton}} &= 1.67 \times 10^{-27} \text{ kg} \\
h &= 6.64 \times 10^{-34} \text{ J} \\
\pi &= 3.14 \\
c &= 3.00 \times 10^8 \text{ m/s} \\
\int x\sqrt{x^2 \pm a^2} \, dx &= \frac{1}{3} \sqrt{(x^2 \pm a^2)^3} \\
\int \frac{dx}{\sqrt{x^2 \pm a^2}} &= \frac{\pm x}{a^2 \sqrt{x^2 \pm a^2}} \\
\int e^{-ax} \, dx &= -\frac{1}{a} e^{-ax} \\
\int \frac{xdx}{(x^2 \pm a^2)^{3/2}} &= -\frac{1}{\sqrt{x^2 \pm a^2}} \\
\int \frac{xdx}{\sqrt{a^2 - x^2}} &= -\sqrt{a^2 - x^2} \\
\int \frac{dx}{x\sqrt{x^2 \pm a^2}} &= \frac{\ln\left(x + \sqrt{x^2 \pm a^2}\right)}{a} \\
\int \frac{dx}{x\sqrt{x^2 + a^2}} &= \frac{\ln\left(a + \sqrt{x^2 + a^2}\right)}{x} \\
\int \frac{dx}{\sqrt{x^2 \pm a^2}} &= \ln\left(x + \sqrt{x^2 \pm a^2}\right), \quad \text{or} \quad -\cos^{-1}\left(\frac{x}{a}\right) \\
\int \frac{dx}{a + bx} &= \frac{1}{b} \ln(a + bx) \\
\int \frac{dx}{(a + bx)^2} &= -\frac{1}{b(a + bx)}
\end{align*}
A Van de Graaff generator is a device for generating a large electric potential by building up charge on a hollow metal sphere. A typical classroom-demonstration model has a diameter of 30 cm.

(a)  [10 pts.] How much charge is needed on the sphere for its potential to be 500 kV?
(b)  [10 pts.] What is the electric field strength just inside and just outside the surface of the sphere when it is charged to 500 kV?
Show all work!! Report all numbers to three (3) significant figures.

A proton (charge $1.602 \times 10^{-19}$ C) is fired with a speed of $1.0 \times 10^6$ m/s through the 10 μF parallel-plate capacitor shown in the figure. The voltage across the plates of the capacitor is 50 kV, and the distance between the plates is 10 cm.

(a) [10 pts.] Find the magnitude of the electric field between the capacitor’s plates.

(b) [10 pts.] Find the electric force on the proton inside the capacitor.

(c) [10 pts.] What magnetic field, both magnitude and direction, must be applied to allow the proton to pass through the capacitor with no change in speed or direction?
Show all work!! Report all numbers to three (3) significant figures.

The switch in the circuit shown in the figure has been closed for a long time.

(a) [10 pts.] What is the current in each leg of the circuit?

(b) [10 pts.] When the switch is opened, the current in the inductor drops by a factor of 2 in 8 μs. What is the value of the inductance?
Show all work!! Report all numbers to three (3) significant figures.

The figure is a cross section through three long wires with linear mass density 50.0 g/cm. They each carry the same current $I$ in the directions shown. The lower wires are 4.00 cm apart and are attached to a table. The upper wire “floats” so as to form an equilateral triangle with the lower wires.

(a)  [10 pts.] What current $I$ allows the upper wire to “float”?
(b)  [10 pts.] What are the magnitude and direction of the magnetic field vector due to the lower wires at the location of the upper wire?
Show all work!! Report all numbers to three (3) significant figures.

A method to measure inductance is to connect the inductor in series with a known capacitance $C$, a known resistance $R$, an AC ammeter, and a variable-frequency signal generator. The frequency of the signal generator is varied and the emf $\mathcal{E}_{\text{rms}}$ is kept constant until the current $I$ is maximum.

(a) [10 pts.] If $C = 10 \ \mu F$, $\mathcal{E}_{\text{rms}} = 10 \ \text{V}$, $R = 100 \ \Omega$, and $I$ is maximum at the frequency $f = 800 \ \text{Hz}$, what is the inductance $L$?

(b) [10 pts.] What is the electric power dissipated in the circuit when $I$ is maximum?
Show all work!! Report all numbers to three (3) significant figures.

An object located 100 cm from a concave spherical mirror forms a real image 75 cm from the mirror.

(a) \[10 \text{ pts.}\] Find the focal length of the concave mirror (including its sign).

The mirror is now turned around so that its convex side faces the object. The mirror is moved so that the image is now 35 cm behind the mirror.

(b) \[10 \text{ pts.}\] How far was the mirror moved? Was it moved toward or away from the object?

(c) \[10 \text{ pts.}\] Sketch a principal ray diagram for the object and the convex mirror after it is moved. Be as accurate as you can.
A positive charge \( +Q = 4.0 \text{ nC} \) is located at \( x = -a \), where \( a = 0.20 \text{ m} \).

(a) [10 pts.] A second equal positive charge \( +Q \) is brought from infinity to \( x = +a \). What is the change of electric potential energy of the second charge?

(b) [10 pts.] With the two equal positive point charges at \( x = -a \) and \( x = +a \), a third charge \( -Q \) is brought from infinity to the origin. What is the change of electric potential energy of the third charge?
Show all work!! Report all numbers to three (3) significant figures.

A 10 cm by 5 cm rectangular loop with resistance 2.5 Ω is pulled through a 20 cm by 20 cm region of uniform magnetic field $B = 1.7 \, T$ (see figure) with constant speed $v = 2.4 \, \text{cm/s}$. The front of the loop enters the region of the magnetic field at time $t = 0$.

(a) [10 pts.] Find and graph the magnetic flux through the moving loop as a function of time from $t = 0$ to $t = 16 \, \text{s}$ (let a flux out of the page be positive).

(b) [10 pts.] Find and graph the induced current in the moving loop as a function of time from $t = 0$ to $t = 16 \, \text{s}$ (let a counterclockwise current be positive).
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[10 pts.] The figure is a graph of the light intensity on a screen behind a double slit. The slit spacing is 0.20 mm and the screen is 2.0 m behind the slits. What is the wavelength (in nm) of the light?
A laser beam of small but finite radius $r$ passes through a thin diverging lens with a focal length of $-10$ cm. The laser beam’s axis is on the principal axis of the lens.

(a) \([10 \text{ pts.}]\) At what distance past the lens has the laser beam’s intensity decreased by a factor of 4?

(b) \([10 \text{ pts.}]\) At what distance past the lens has the laser beam’s electric field strength decreased by a factor of 4?
The figure shows a circular current loop near a wire, carrying currents of 0.20 A and 2.0 A, respectively, in the directions indicated.

(a) [10 pts.] What is the magnitude and direction of the torque on the current loop?

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[10 pts.] A charge of 2 μC is 20 cm above the center of a square of side length 40 cm. Find the electric flux through the square. (Hint: Do not integrate.)
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(a) [10 pts.] How much charge is needed on the sphere for its potential to be 500 kV?

(b) [10 pts.] What is the electric field strength just inside and just outside the surface of the sphere when it is charged to 500 kV?

\[ V = \frac{Q}{4\pi \varepsilon_0 R} \]

\[ E_{\text{inside}} = 0 \]

\[ E_{\text{outside}} = \frac{Q}{4\pi \varepsilon_0 R^2} = \frac{3.73 \times 10^6 \text{ V}}{\varepsilon_0} \]

Outside it can be treated as a point charge.
Show all work!! Report all numbers to three (3) significant figures.

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(c) [10 pts.] What magnetic field, both magnitude and direction, must be applied to allow the proton to pass through the capacitor with no change in speed or direction?

\[
E = \frac{\Delta V}{d} = \frac{50 \times 10^3 \text{ V}}{1 \text{ m}} = 5 \times 10^5 \text{ V/m} \quad \text{or} \quad 5 \times 10^5 \text{ N/C}
\]

\[
F = qE = (1.602 \times 10^{-19} \text{ C})(5 \times 10^5 \text{ N/C}) = 8.01 \times 10^{-14} \text{ N}
\]

For the magnetic field to be applied, it must be equal to the electric force. Magnitudes equal between forces:

\[
qUB = qE
\]

\[
B = \frac{E}{U} = \frac{5 \times 10^5 \text{ N/C}}{1 \times 10^6 \text{ m/s}} = 0.5 \text{ T}
\]
The switch in the circuit shown in the figure has been closed for a long time.

(a) [10 pts.] What is the current in each leg of the circuit?

(b) [10 pts.] When the switch is opened, the current in the inductor drops by a factor of 2 in 8 μs. What is the value of the inductance?

(a) After a long time the current is steady \( \Rightarrow \frac{dI}{dt} = 0 \) in all legs.

Since the potential drop across the inductor is:
\[
\Delta V_L = L \frac{dI}{dt} = L \cdot 0 = 0
\]

\( \Rightarrow \Delta V_{24Ω} = 0 \) (since the 24Ω resistor is in parallel with inductor and \( \Delta V_{24Ω} = \frac{\Delta V_{24Ω}}{24Ω} = 0 \))

\( \Rightarrow \frac{I_{24Ω}}{24Ω} = 0 \)

So all the current goes through the 5kΩ resistor and the inductor with the value
\[
I = \frac{E}{R} = \frac{12V}{5000Ω} = 2.40 \times 10^{-3} A
\]

(b) \( I = I_0 e^{-t/\tau} \) for a discharging RL circuit, \( \tau = \frac{L}{R} \).

\[
\frac{I}{I_0} = e^{-t/\tau} \Rightarrow \tau = \frac{L}{R} = \frac{(8 \times 10^{-6})}{(24Ω)} (24Ω) = 277 \mu H
\]
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(a) [10 pts.] If $C = 10 \mu F$, $E_{rms} = 10$ V, $R = 100$ $\Omega$, and $I$ is maximum at the frequency $f = 800$ Hz, what is the inductance $L$?

(b) [10 pts.] What is the electric power dissipated in the circuit when $I$ is maximum?

\[ Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \]

When $I$ is at maximum, $\omega L - \frac{1}{\omega C} = 0 \Rightarrow \omega = \frac{1}{\sqrt{LC}}$

\[ 2\pi f = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{(2\pi f)^2 C} \approx 3.96 \times 10^{-3} H \]

\[ L = 3.96 \text{ mH} \]

\[ P = E_{rms} \cdot I_{rms} = \frac{E_{rms}}{Z} = \frac{E_{rms}}{R} \]

\[ P = \frac{E_{rms}^2}{R} \approx 1 \text{ W} \]

At maximum, phasors of voltage & current are aligned, thus $P = V \times I$ (No cosine)
Show all work!! Report all numbers to three (3) significant figures.

An object located 100 cm from a concave spherical mirror forms a real image 75 cm from the mirror.

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(c) [10 pts.] Sketch a principal ray diagram for the object and the convex mirror after it is moved. Be as accurate as you can.

\[
\begin{align*}
\frac{1}{\sigma} + \frac{1}{i} &= \frac{1}{f} \\
\frac{1}{100\text{cm}} + \frac{1}{75\text{cm}} &= \frac{1}{f} \\
f &= \frac{100\times75}{100+75} \text{ cm} = 42.85 \text{ cm}
\end{align*}
\]

\[
\begin{align*}
\frac{1}{\sigma'} + \frac{1}{i'} &= \frac{1}{f'} \\
\frac{1}{-35\text{cm}} + \frac{1}{-42.85\text{cm}} &= \frac{1}{f'} \\
f' &= -f = -42.85 \text{ cm}
\end{align*}
\]

\[
\begin{align*}
\sigma' &= \frac{i' \cdot f'}{i' - f'} = \frac{(-35\text{cm}) \times (-42.85\text{cm})}{(-35\text{cm}) - (-42.85\text{cm})} = 190.9 \text{ cm}
\end{align*}
\]

\[
\begin{align*}
\Delta \sigma &= \sigma' - \sigma = 90.9 \text{ cm}
\end{align*}
\]

The mirror was moved by AWAY!

\[
\begin{align*}
\sigma &= 100 \text{ cm} \\
i &= 75 \text{ cm}
\end{align*}
\]
Show all work!! Report all numbers to three (3) significant figures.

A positive charge $+Q = 4.0 \text{ nC}$ is located at $x = -a$, where $a = 0.20 \text{ m}$.

(a) [10 pts.] A second equal positive charge $+Q$ is brought from infinity to $x = +a$. What is the change of electric potential energy of the second charge?

$$U = \frac{Q^2}{4\pi \epsilon_0} \frac{1}{2a} = \frac{4 \cdot (4 \cdot 10^{-3} \text{ C})^2}{4\pi \left(8.85 \times 10^{-12}\right) \cdot 2 \cdot (0.2 \text{ m})} = 3.60 \times 10^{-7} \text{ J}$$

(b) [10 pts.] With the two equal positive point charges at $x = -a$ and $x = +a$, a third charge $-Q$ is brought from infinity to the origin. What is the change of electric potential energy of the third charge?

$$U = -\frac{Q}{4\pi \epsilon_0} \left[ \frac{Q}{a} + \frac{Q}{a} \right] = -\frac{(4 \cdot 10^{-3} \text{ C})^2 \cdot 2}{4\pi \left(8.85 \times 10^{-12}\right) (0.2 \text{ m})} = -1.44 \times 10^{-6} \text{ J}$$
**Show all work!! Report all numbers to three (3) significant figures.**

A 10 cm by 5 cm rectangular loop with resistance 2.5 Ω is pulled through a 20 cm by 20 cm region of uniform magnetic field $B = 1.7$ T (see figure) with constant speed $v = 2.4$ cm/s. The front of the loop enters the region of the magnetic field at time $t = 0$.

(a) **[10 pts.]** Find and graph the magnetic flux through the moving loop as a function of time from $t = 0$ to $t = 16$ s (let a flux out of the page be positive).

(b) **[10 pts.]** Find and graph the induced current in the moving loop as a function of time from $t = 0$ to $t = 16$ s (let a counterclockwise current be positive).

\[ \Phi_{\text{magn}} = B \cdot A = (1.7 \text{ T}) \cdot (10 \text{ cm} \cdot 5 \text{ cm}) = (1.7 \text{ T}) \cdot (0.1 \text{ m} \cdot 0.05 \text{ m}) \]

\[ 85 \text{ T} \cdot \text{cm}^2 = 8.5 \cdot 10^{-3} \text{ T} \cdot \text{m}^2 \]

\[ I = \frac{V}{R} \]

\[ V_{\text{ind}} = \frac{d\Phi}{dt} = \frac{8.5 \cdot 10^{-3} \text{ T} \cdot \text{m}^2}{4.16 \text{ s}} \]

\[ I = \frac{2.04 \cdot 10^{-3} \text{ V}}{2.5 \Omega} \]

\[ = 8.16 \cdot 10^{-4} \text{ A} \]
Show all work!! Report all numbers to three (3) significant figures.

[10 pts.] The figure is a graph of the light intensity on a screen behind a double slit. The slit spacing is 0.20 mm and the screen is 2.0 m behind the slits. What is the wavelength (in nm) of the light?

\[ d \sin \theta_m = m \lambda \] (Angle of first max after the central max)

\[ y = L \sin \theta \]

\[ \Rightarrow y_m = \frac{m \lambda}{d} L \] (Position of the max)

\[ \Rightarrow \lambda = \frac{y_1 \cdot d}{L} = \frac{(0.5 \, \text{cm}) \cdot (0.20 \times 10^{-5} \, \text{m})}{2.0 \, \text{m}} \]

\[ \lambda = 5.00 \times 10^{-7} \, \text{m} \]

\[ \lambda = 500 \, \text{nm} \]
A laser beam of small but finite radius \( r \) passes through a thin diverging lens with a focal length of -10 cm. The laser beam’s axis is on the principal axis of the lens.

(a) [10 pts.] At what distance past the lens has the laser beam’s intensity decreased by a factor of 4?

\[ \text{Intensity decreased by a factor of 4} \]

(b) [10 pts.] At what distance past the lens has the laser beam’s electric field strength decreased by a factor of 4?

\[ \text{Electric field strength decreased by a factor of 4} \]

---

**Solution:**

#### (a) Intensity Decrease

1. **Beam incoming as parallel rays:**
   
   ![Beam Diagram](image)

2. **Intensity formula:**
   
   \[ I = \frac{\text{power}}{\text{area}} = \frac{\text{power}}{\pi r^2} \]

3. **Distance formula:**
   
   \[ \frac{I}{I_0} = \frac{I}{I_0} = \frac{\pi r^2}{\pi R^2} = \left(\frac{r}{R}\right)^2 \]

4. **Similar triangles:**
   
   \[ \frac{r}{R} = \frac{10\text{ cm}}{x+10\text{ cm}} \]

5. **Solve for \( x \):**
   
   \[ \frac{1}{4} = \left(\frac{10}{x+10}\right)^2 \]
   
   \[ x = 10\text{ cm} \text{ past the lens} \]

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#### (b) Electric Field Strength Decrease

1. **Electric field strength formula:**
   
   \[ \frac{I}{I_0} = \frac{|E|^2}{|E_0|^2} = \frac{1}{16} \]

2. **Solve for \( x \):**
   
   \[ \frac{1}{16} = \left(\frac{10}{x+10}\right)^2 \]
   
   \[ x = 30\text{ cm} \text{ past the lens} \]
Show all work!! Report all numbers to three (3) significant figures.

The figure shows a circular current loop near a wire, carrying currents of 0.20 A and 2.0 A, respectively, in the directions indicated.

(a) [10 pts.] What is the magnitude and direction of the torque on the current loop?
(b) [10 pts.] What is the loop's orientation that minimizes the loop energy in the magnetic field of the wire? Specify the counterclockwise angle from the upward vertical (dashed line in the figure).

\[ \vec{\tau} = \vec{\mu} \times \vec{B} \]

\[ \vec{\tau} = \frac{\mu_0 I \pi r^2}{2\pi} \times \vec{B} \]

\[ \vec{\tau} = (2\pi \times (0.02 \text{ m})^2) \times (-\hat{x}) \]

\[ \vec{\tau} = (6.28 \times 10^{-3}) (-\hat{x}) \]

\[ \vec{B} = \frac{\mu_0 I}{2\pi r} \]

\[ \vec{B} = \frac{(4\pi \times 10^{-7}) (2.0 \text{ A}) \hat{\gamma}}{2\pi (0.02 \text{ m})} \]

\[ \vec{B} = 2 \times 10^{-5} \hat{\gamma} \]

\[ \vec{E} = (6.28 \times 10^{-3}) (2 \times 10^{-5}) (-\hat{x} \times \hat{\gamma}) = 1.26 \times 10^{-10} \hat{\gamma} \]

(b) \[ E = -\vec{\mu} \cdot \vec{B} \]

So, \( \mu \) will be min. \( M \) of \( B \) is max. \( \vec{B} \) is min. \( \vec{B} \) max.

Which is zero degrees from the vertical.
Show all work!! Report all numbers to three (3) significant figures.

[10 pts.] A charge of 2 $\mu$C is 20 cm above the center of a square of side length 40 cm. Find the electric flux through the square. (Hint: Do not integrate.)

\[ \Phi_{\text{total}} = E \cdot A_{\text{closed sphere}} \]

\[ A = 4 \pi r^2 \]

\[ E = \frac{Q}{4 \pi \varepsilon_0 r^2} \]

\[ \Phi_{\text{total}} = \frac{Q}{\varepsilon_0} = 0.226 \times 10^{-6} \text{ Nm}^2/\text{C} \]

\[ \Phi_{\text{side}} = \frac{\Phi_{\text{total}}}{6} = \frac{0.017 \times 10^6}{6} = 0.226 \times 10^6 \text{ Nm}^2/\text{C} \]

\[ 3 \times 10^3 \text{ Nm}^2/\text{C} \]