Physics 2220  Third Exam  Name: SOLUTIONS  
Summer 2018  (Chs 31–34)  Rohit Kumar (WEB L114)  
Friday 20 July  Circle your Discussion TA:  Mandefro Teferi (WEB L112)  
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You may use your one sheet of notes and formulas, but do not collaborate with any other person. Do all four problems, showing your method and working clearly (a right answer alone is not necessarily enough). Include correct SI units in your answers where appropriate. The number of marks for each portion of a question is given in square brackets, [ ], to its right.

1. (a) For each of the cases (i) and (ii) illustrated at the right, give the direction of the induced current in the circular loop, writing cw for clockwise, ccw for counterclockwise, or none if there will be no induced current.  [2]  

(i)  

(ii)  

(b) In the sketch, a 12.0-cm metal rod of resistance 4.00 Ω moves with a speed of 15.0 m/s at right angles to a uniform 0.800-T magnetic field that points perpendicularly out of the paper. Calculate:

(i) the motional emf, $\varepsilon$, developed in the rod, stating which of the rod’s ends is at the higher potential;  [5]  

$\varepsilon = Blv = (0.800)(0.12)(15) = 1.44 \text{ V}$  

The right-hand end is at the higher potential.  

(ii) the electric field in the rod, giving both its magnitude and direction;  [3]  

$E = \frac{\varepsilon}{L} = \frac{1.44 \text{ V}}{0.12 \text{ m}} = 12 \text{ V/m}$, pointing from right to left in the rod.  

(iii) the electric current in the rod.  [1]  

The current is zero, since there is no complete circuit.  

(c) A square coil of side 5.00 cm contains 120 turns of wire and is positioned perpendicular to a uniform 0.600-T magnetic field. It is then quickly and uniformly pulled out of the magnetic field (moving perpendicularly to $B$) into a region where B drops abruptly to zero, as in the diagram at the right. It takes 0.100 s for the whole coil to reach the field-free region.

(i) Calculate the current induced in the coil and the total thermal energy produced during the 0.100-s interval, given that the resistance of the coil is 200 Ω.  [8]  

(ii) Once the right-hand edge of the coil is free of the field, find the magnetic force acting on its left-hand edge, and the work done in pulling the coil out of the field.  [5]  

(iii) Are the answers to (i) and (ii) physically consistent with each other?  [2]
(c) (i) \[ |E| = N \frac{d\Phi}{dt} = N \frac{\Delta \Phi}{\Delta t} = \frac{NBA}{0.10} = \frac{(120)(0.600)(0.05)^2}{0.10} = 1.8 \text{ V} \]

\[ I = \frac{|E|}{R} = \frac{1.8 \text{ V}}{200 \Omega} = 0.0090 \text{ A} = 9.0 \text{ mA} \]

The total thermal energy developed in time \( \Delta t \) is \( I^2 R \Delta t = (9.0 \times 10^{-3})^2 (200)(0.10) = 0.00162 \text{ J} \), or \( 1.62 \text{ mJ} \), since there are \( N \) turns of wire.

(ii) \[ F = I(NL)B, \text{ and the work done is } W = F \Delta x, \text{ or} \]

\[ W = I N L B \Delta x = (9.0 \times 10^{-3})(120)(0.05)(0.60)(0.05) = 1.62 \text{ mJ} \]

(iii) Yes, the answers are the same, as expected. It is the work done by the agent pulling the coil from the magnetic field that ultimately provides the thermal energy in the wires.
2. (a) The figure at the right shows a circuit with a battery, two identical resistors, an inductor, and a switch, $S$. Is the current through the central resistor (that is, the ‘vertical’ one) more than, less than, or the same as the other resistor? \[8\]

(i) just after the closing of switch $S$;
- more (the inductor acts like a break in the circuit)

(ii) a long time after the closing of $S$;
- same (inductor acts like a wire, so the two resistors are effectively in parallel)

(iii) immediately following the reopening of $S$ after it has been closed for a long time;
- same (the resistors now are in series)

(iv) a long time after the reopening of $S$.
- same (both are zero)

(b) How many time constants will it take for the decaying current in an $LR$ circuit to be reduced to 5.0 per cent of its original value $I_0$? \[3\]
\[
I = I_0 e^{-t/\tau} \Rightarrow 0.05 I_0 = I_0 e^{-t/\tau} \Rightarrow 0.05 = e^{-t/\tau} \Rightarrow -\frac{t}{\tau} = \ln(0.05) \Rightarrow t = -\tau \ln(0.05) = \left[\frac{3.0 \tau}{3 \text{ time constants}}\right]
\]

(c) For oscillations of an $LC$ circuit in which $C = 4.00 \mu F$, the maximum potential difference across the capacitor during the oscillations is 1.50 V, and the maximum current through the inductor is 50.0 mA. Calculate:

(i) the value of the inductance, $L$; \[5\]

(ii) the frequency, $f$, of the oscillations; \[2\]

(iii) the time needed for the charge on the capacitor to increase from zero to its maximum value. \[1\]

(d) Consider an $LC$ circuit in which the capacitor is uncharged at time $t = 0$. In terms of the period $T$ of the $LC$ oscillations, calculate the first time after $t = 0$ at which the energy stored in the capacitor will be increasing at its greatest rate. Also, express that greatest rate of increase in terms of $T$, $C$, and $Q_{\text{max}}$. (The identity $\sin 2x = 2 \sin x \cos x$ may be helpful.) \[9\]

(i) The maximum energies in the inductor and the capacitor are equal, so
\[
\frac{1}{2} L I_{\text{max}}^2 = \frac{1}{2} C V_{\text{max}}^2 \Rightarrow L = \frac{C V_{\text{max}}^2}{I_{\text{max}}^2} = \frac{(4.0 \times 10^{-6})(1.5)^2}{(50 \times 10^{-3})^2} = 0.0036 \text{ H},
\]
or
\[
L = 3.60 \text{ mH}
\]

(ii) \[
2\pi f = \omega = \frac{1}{\sqrt{LC}} \Rightarrow f = \frac{1}{2\pi} \sqrt{\frac{1}{LC}} = \frac{1}{2\pi} \sqrt{\frac{1}{(3.6 \times 10^{-3})(4.0 \times 10^{-6})}} = 1326.3 \text{ Hz},
\]
or
\[
f = 1.33 \text{ kHz}
\]

(iii) The time needed is one-fourth of a full period, or \[
\frac{1}{4} T = \frac{1}{4} \left(\frac{1}{f}\right) = \frac{1}{4f} = \frac{1}{4(1326.3)} \approx 0.000188, \text{ or } 0.188 \text{ ms}
\]

\[\text{or } 188 \mu s\]
(d) Because the capacitor is uncharged at \( t = 0 \), the sinusoidally varying charge on the capacitor is given by \( Q(t) = Q_{\text{max}} \sin \omega t \). The energy stored in the capacitor is \( U = \frac{Q^2}{2C} \), or
\[
U = \frac{Q_{\text{max}}^2 \sin^2 \omega t}{2C}.
\]
Using the chain rule, the rate at which this energy is changing is
\[
\frac{dU}{dt} = \frac{Q_{\text{max}}^2}{2C} 2 \sin \omega t \cos \omega t,
\]
which is
\[
\frac{dU}{dt} = \frac{Q_{\text{max}}^2 \sin 2\omega t}{2C}.
\]
This will have its greatest positive value when \( 2\omega t = \frac{\pi}{2} \Rightarrow t = \frac{\pi}{4\omega} \).

Since \( \omega = \frac{2\pi}{T} \), we have \( t = \frac{\pi}{4 \left( \frac{2\pi}{T} \right)} \Rightarrow t = \frac{T}{8} \), and the greatest rate of increase of \( U \) (i.e., the maximum of \( \frac{dU}{dt} \)) will be
\[
\frac{\omega Q_{\text{max}}^2}{2C} = \frac{2\pi}{T} \frac{Q_{\text{max}}^2}{2C} = \frac{\pi Q_{\text{max}}^2}{TC}.
\]
3. (a) When a certain sinusoidal AC voltage is applied across a 700-Ω resistor, electrical energy is transformed into thermal energy in the resistor at an average rate of 7.0 W. What are the values of $V_{\text{rms}}$ and $V_{\text{max}}$ in this case? [6]

(b) An ideal transformer has $N_1 = 500$ primary turns and $N_2 = 10$ secondary turns.

(i) If the secondary voltage $V_2$ is 120 V (rms), what is $V_1$ (rms)? [2]

(ii) If the secondary is connected to a resistive load $R_L = 15$ Ω, what will be the (rms) currents $I_2$ and $I_1$ in the secondary and the primary respectively? [5]

(iii) What is $R_{\text{eq}}$, the equivalent resistance of the load resistance when viewed from the primary side? [3]

(c) Suppose that 48.0 kW of power is to be transmitted from a generating station to a town over transmission lines having resistance 2.00 Ω. Using ideal transformers (i.e., those that are 100 per cent efficient), the voltage is initially stepped up to some value $V$ for transmission and is then stepped down to 120 V at the town. Calculate the percentage loss of power as thermal energy in the transmission lines if $V$ is chosen to be: (i) 480 V; (ii) 9600 V; (iii) 240 V. [10]

(a) Power $P = \frac{V^2}{R}$, so $P_{\text{avg}} = P = \frac{V^2}{R} = \frac{V_{\text{rms}}^2}{R}$. Then

$V_{\text{rms}} = \sqrt{R \cdot P} = \sqrt{(700) \cdot (7)} = \sqrt{4900} \Rightarrow V_{\text{rms}} = 70$ V

For sinusoidal AC, $V_{\text{rms}} = \frac{V_{\text{max}}}{\sqrt{2}} \Rightarrow V_{\text{max}} = \sqrt{2} \cdot V_{\text{rms}} = \sqrt{2} \cdot (70) = 99$ V.

(b) (i) $V_1 = \left(\frac{N_1}{N_2}\right) V_2 = \left(\frac{500}{10}\right) (120) = 50 \cdot (120) \Rightarrow V_1 = 6000 \text{ V}$.

(ii) $I_2 = \frac{V_2}{R_L} = \frac{120}{15} \Rightarrow I_2 = 8.0 \text{ A}$. For an ideal transformer,

$I_1 V_1 = I_2 V_2$, so $I_1 = \frac{I_2 V_2}{V_1} = \frac{(8)(120)}{6000} = \frac{8}{50} = 0.16 \Rightarrow I_1 = 0.16 \text{ A}$.

(iii) $R_{\text{eq}} = \left(\frac{N_1}{N_2}\right)^2 R_L = \left(\frac{500}{10}\right)^2 (15) = (50)^2 (15) \Rightarrow R_{\text{eq}} = 37500 \Omega$.

(Alternatively, $R_{\text{eq}} = \frac{V_2}{I_2} = \frac{6000 \text{ V}}{0.16 \text{ A}} = 37.5 \text{ k}\Omega$.)
3. (cont’d) \hspace{1cm} \text{Name: SOLUTIONS}

\( (c) \quad P = IV \Rightarrow I = \frac{P}{V} \), and the power lost to thermal form is

\[ I^2R = \left(\frac{P}{V}\right)^2R = \left(\frac{48000}{V}\right)^2. \]

(i) For \( V = 480 \, \text{V} \), power loss = \((\frac{48000}{480})^2(2) = (100)^2(2) = 20000 \, \text{W}, \)
which is \( \frac{20000}{48000} \times 100 \% = 41.7 \, \text{per cent} \) (unacceptably large).

(ii) For \( V = 9600 \, \text{V} \), power loss = \((\frac{48000}{9600})^2(2) = (5)^2(2) = 50 \, \text{W}, \)
which is only \( \frac{50}{48000} \times 100 \% = 0.1 \, \text{per cent} \) (very small).

This illustrates why it is important to transmit power at high voltages.

(iii) For \( V = 240 \, \text{V} \), the power loss would be \( \left(\frac{48000}{240}\right)^2(2) = (200)^2(2) = 80000 \, \text{W}, \) which exceeds the power we need to transmit. That is, all the power would have been lost in this case, meaning transmission at 240V is not even possible.
4. Consider a radio transmitter that can be thought of as a point source broadcasting sinusoidal electromagnetic waves uniformly in all directions. At 1.50 km from the transmitter, the amplitude of the radio wave’s electric field is $E_{\text{max}} = 3.50 \text{ V/m}$.

(The permittivity of vacuum is $\varepsilon_0 \approx 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$, the permeability of vacuum is $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m} / \text{A}$, and the speed of light in vacuum is $c = 1/\sqrt{\varepsilon_0 \mu_0} \approx 3.00 \times 10^8 \text{ m/s}$.)

(a) What is the power output of the transmitter? [8]

(b) At a distance of 10.0 km from the transmitter, calculate:
   (i) the intensity of the radio waves; [3]
   (ii) the value of $E_{\text{max}}$; [4]
   (iii) the amplitude of the magnetic field, $B_{\text{max}}$. [2]

(c) At a certain point P in space far from the transmitter, energy is being propagated in the positive x-direction by the radio waves. If at a given instant the magnetic field vector $\mathbf{B}$ for one of those waves points in the positive y-direction at P, what is the instantaneous direction of its electric field vector $\mathbf{E}$?

\[
\text{(a) intensity } \mathcal{I} = \frac{E_{\text{max}}^2}{2\mu_0 c} = \frac{P}{4\pi r^2} \quad \Rightarrow \quad P = \frac{4\pi r^2 E_{\text{max}}^2}{2\mu_0 c}, \quad \text{or}
\]

\[
P = \frac{4\pi (1500)^2 (3.5)^2}{2(4\pi \times 10^{-7}) c} = \frac{(1500)^2 (3.5)^2}{(2\times10^{-7})(3\times10^8)} \approx 459 \text{ kW}
\]

\[
\text{(b)(i) } \mathcal{I} = \frac{P}{4\pi r^2} \approx \frac{459 \text{ kW}}{4\pi (10000)^2} \approx \frac{3.66 \times 10^{-4} \text{ W}}{\text{m}^2}
\]

\[
\text{(ii) } \mathcal{I} = \frac{E_{\text{max}}^2}{2\mu_0 c} \Rightarrow E_{\text{max}} = \sqrt{2\mu_0 c \mathcal{I}} = \sqrt{2(4\pi \times 10^{-7})(3\times10^8)(3.66\times10^{-4})}
\]

\[
\Rightarrow E_{\text{max}} \approx 0.525 \frac{\text{V}}{\text{m}}
\]

\[
\text{(iii) } B_{\text{max}} = \frac{E_{\text{max}}}{c} \approx \frac{0.525}{3.0 \times 10^8} \approx 1.75 \times 10^{-9} \text{ T}, \text{ or } 1.75 \mu \text{T}
\]

(c) Since $\mathbf{S} = \mu_0 \mathbf{E} \times \mathbf{B}$ points in the $\hat{i}$ direction, and $\mathbf{B}$ is in the $\hat{j}$ direction,

we conclude that $\mathbf{E}$ must point in the direction $-\hat{k}$ or the negative z-direction.

\[
\begin{array}{ccc}
\mathbf{E} & \rightarrow x & \rightarrow \hat{i} \\
\mathbf{E} \times \mathbf{B} & \rightarrow z & \rightarrow \mathbf{E} \\
\end{array}
\]