1. (a) A 1.0-μF capacitor having an initial stored electrostatic potential energy of 0.50 J is discharged through a 1.0-MΩ resistor.

   (i) What is the initial charge on the capacitor? [4]

   (ii) What is the current through the resistor when the discharge has just begun? [5]

(b) How to make a flashing lamp: The diagram at the right shows a fluorescent lamp L connected in parallel across the 0.15 μF capacitor of an RC circuit with a 95-V battery and a large resistance R. Current passes through the lamp only when the potential difference across it reaches the breakdown value of 72 V. Then the capacitor discharges essentially instantaneously through the lamp, causing it to flash. If we want the lamp to flash two times per second, what should be the value of R? [11]

   \[
   (a) \quad U_o = \frac{Q_o^2}{2C} \Rightarrow Q_o = \sqrt{2Cu_o} = \sqrt{2 \left(1.0 \times 10^{-6}\right)(0.5)} \Rightarrow Q_o = 1.0 \times 10^{-3} C = 1.0 \text{ mC}.
   \]

   \[
   (i) \quad Q(t) = Q_o e^{-t/RC}, \quad \text{and} \quad I(t) = -\frac{dQ}{dt} = \frac{Q_o e^{-t/RC}}{RC},
   \]

   \[
   \text{so at} \quad t = 0, \quad I = \frac{Q_o}{RC} = \frac{1.0 \times 10^{-3}}{(1.0 \times 10^{-6})(1.0 \times 10^{-6})} \Rightarrow I = 1.0 \text{ mA}.
   \]

   \[
   (b) \quad V(t) = E \left(1 - e^{-t/RC}\right), \quad \text{as in equation (28.14)}. \quad \therefore
   \]

   \[
   \frac{V}{E} = 1 - e^{-t/RC}, \quad \text{or} \quad e^{-t/RC} = 1 - \frac{V}{E} = 1 - \frac{72}{95} \approx 0.2421. \quad \text{In order to flash twice per second, the time} \ t \ \text{needed to reach} \ 72 \text{V} \ \text{must be} \ t = 0.55. \quad \text{That is,} \quad e^{-0.55/RC} = 0.2421, \ \text{or} \ e^{-t/RC} \approx 0.2421. \ \text{Then}
   \]

   \[
   -\frac{1}{2RC} = \ln(0.2421) \Rightarrow R = \frac{-1}{2 \ln(0.2421)(0.15 \times 10^{-6})} \approx 2.35 \times 10^6 \Omega, \ \text{or}
   \]

   \[
   R = 2.35 \text{ MΩ}.
   \]
2. (a) Calculate the lateral shift \( d \) between the (parallel) incident and emergent rays when light in air is incident at 37.6° from the normal on a rectangular glass block of width 4.00 cm and refractive index 1.50.

\[
\ell = \frac{4.00 \text{ cm}}{\cos 24^\circ} \approx 4.38 \text{ cm}, \quad \text{and} \quad d = \ell \sin 13.6^\circ, \quad \text{or} \quad d = \left(\frac{4.00 \text{ cm}}{\cos 24^\circ}\right) \sin 13.6^\circ \Rightarrow d = 1.03 \text{ cm}
\]

(b) Light in air is incident at angle \( \theta \) from the normal on the left-hand face of a glass prism of refractive index 1.50 and apex angle 60.0°. The refracted light then travels through the prism and strikes its right-hand face. Calculate the angle of deviation, \( \delta \), between the incident ray and the final ray that emerges into the air in both of the following cases:

(i) \( \theta = 57.0^\circ \) \quad [14];
(ii) \( \theta = 0.0^\circ \). \quad [6]

From the diagram,
\[
\delta = 25^\circ + 15.1^\circ = 38.1^\circ
\]

(ii) If \( \theta = 0 \), there is no refraction at the left-hand face of the prism.

The critical angle for air-glass is \( \theta = \sin^{-1} \left(\frac{1}{1.5}\right) \approx 41.8^\circ \), which is less than the 60° angle of incidence at the right-hand face of the prism. Therefore, light is totally internally reflected there and emerges at right angles to the base of the prism (again, no refraction). So in this case \( \delta = 60^\circ \).
3. (a) A large concave spherical mirror has radius of curvature 30.0 cm. An object 7.00 cm tall is placed 20.0 cm in front of the mirror. Describe, as completely as possible, (i) the location, (ii) the size, and (iii) the nature (real or virtual, upright or inverted) of the image.

(b) A thin lens (sketched at the right, though not to scale) is constructed from glass of refractive index 1.50 and is surrounded by air. The magnitudes of the radii of curvature of the left-hand and right-hand surfaces of the lens are, respectively, 10.0 cm and 5.00 cm. Calculate the focal length of the lens.

(c) An object is placed 30 cm in front of a converging lens of focal length 20 cm, and a convex mirror whose radius of curvature has magnitude 120 cm is placed 40 cm behind the lens. Locate and describe the characteristics of the final image as completely as possible, including its magnification. Where would one’s eye have to be placed in order to see this image?

(a) The focal length of a concave spherical mirror is positive, so $f = \frac{R}{2} = \frac{30\text{cm}}{2} \Rightarrow f = 15\text{cm}$. The mirror equation $\frac{1}{f} + \frac{1}{q} = \frac{1}{p}$ gives $\frac{1}{20} + \frac{1}{q} = \frac{1}{15} \Rightarrow \frac{1}{q} = \frac{1}{15} - \frac{1}{20} = \frac{1}{60}$.

$\Rightarrow q = +60\text{cm}$, (real), (inverted)

(i) The image is 60 cm in front of the mirror.

(ii) $M = \frac{-q}{p} = \frac{-60}{20} \Rightarrow M = -3$. Size of image $= 3 \times 7\text{cm} = 21\text{cm tall}$.

(iii) real, inverted

(b) Lens-maker's equation: $\frac{1}{f} = (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) = (1.5-1) \left[ \frac{1}{10} - \frac{1}{5} \right]$ (The centre of curvature is behind the spherical surface in both cases, so the signs of $R_1$ and $R_2$ are positive.)

Then $\frac{1}{f} = 0.5 \left( \frac{1}{10} - \frac{2}{10} \right) = 0.5 \left( -\frac{1}{10} \right) = -\frac{1}{20} \Rightarrow f = -20\text{cm}$.

(ii) object

lens

convex mirror

eye

$\leftarrow 30\text{cm} \rightarrow

\leftarrow 40\text{cm} \rightarrow$
First passage of light through the lens (left to right):
\[
\frac{1}{30} + \frac{1}{q} = \frac{1}{20} \Rightarrow \frac{1}{q} = \frac{3}{60} - \frac{2}{60} = \frac{1}{60} \Rightarrow (q = 60 \text{ cm})
\]
\[
M_1 = -\frac{q}{p} = -\frac{60}{30} \Rightarrow (M_1 = -2)
\]
The image would have been formed 60 cm behind the lens.

Reflection in the convex mirror (of focal length \( f = \frac{-120 \text{ cm}}{2} = -60 \text{ cm} \)):
The (real) image that would have been formed 60 cm behind the lens (i.e., 20 cm behind the mirror) acts as a virtual object (with \( p = -20 \text{ cm} \)) for the mirror:
\[
\frac{1}{20} + \frac{1}{q} = \frac{1}{-60} \Rightarrow \frac{1}{q} = \frac{-1}{60} + \frac{3}{60}
\]
or \( \frac{1}{q} = \frac{2}{60} \Rightarrow (q = 30 \text{ cm}) \), which means that a real image is formed 30 cm in front of (to the left of) the mirror, or 10 cm to the right of the lens, \( M_2 = \frac{-30}{-20} \Rightarrow (M_2 = \frac{3}{2}) \).

Second passage of light thro' the lens (right to left):
\[
\frac{1}{10} + \frac{1}{q} = \frac{1}{20} \Rightarrow \frac{1}{q} = \frac{1}{20} - \frac{2}{20} = -\frac{1}{20} \Rightarrow (q = -20 \text{ cm})
\]
This virtual image is formed 20 cm "in front of" (i.e., to the right of, in this case) the lens, \( M_3 = -\frac{(-20)}{10} \Rightarrow (M_3 = +2) \) (inverted).

The total magnification is \( M = M_1 \times M_2 \times M_3 = (-2) \times (\frac{3}{2}) \times (2) = -6 \).

The final image is virtual. It is between the lens and the mirror, 20 cm to the right of the lens (or 20 cm to the left of the mirror. It is inverted and is 6 times as large as the original object. To see the image, one would need to place one's eye to the left of the lens and look right towards the shiny face of the mirror.
4. (a) A Young's double-slit experiment is conducted with light of wavelength 550 nm, slit width 0.030 mm for each slit, and a centre-to-centre slit separation of 0.150 mm. The distance from the slits to the screen is 1.80 m.

(i) How many complete bright interference fringes appear within the central maximum of the diffraction pattern? \[4\]

(ii) Taking both interference and diffraction effects into account, calculate the intensity of light at the point on the screen whose lateral distance from the centre of the screen is \( y = 2.20 \) mm. Give your answer as a multiple (correct to three significant figures) of the intensity \( I_{\text{max}} \) of the central maximum on the screen (i.e., at \( y = 0 \)). \[5\]

(b) A thin, uniform, transparent coating of material of refractive index 1.60 is deposited on a glass lens of refractive index 1.52. The thickness of the coating is chosen so as to minimize (by means of destructive interference) reflection of normally incident red light of wavelength 672 nm.

(i) Calculate the minimum thickness (greater than zero) needed for the coating. \[5\]

(ii) Will any specific wavelength of light in the visible portion of the spectrum (400 nm to 700 nm) be particularly strongly reflected by this coated lens? If so, what wavelength is it, and what effect will this state of affairs have on the transmission of that particular colour of light through the coated lens? \[5\]

(c) Unpolarized light is incident normally on a set of three successive polarizing discs whose planes are all parallel to each other. The polarizing axis of the first disc is vertical, the axis of the second disc makes an angle \( \theta \) with the vertical, and the axis of the third disc makes an angle \( \theta \) with the axis of the second disc. If the light emerging from the third disc has an intensity that is 38 per cent of the intensity of the unpolarized light incident on the first disc, calculate the value of the angle \( \theta \) correct to the nearest degree. \[7\]

\[\text{(a) (i) } m^{\text{th}} \text{ interference maximum: } d \sin \theta = m \lambda \]

1st diffraction minimum: \( a \sin \theta = \lambda \).

The angles are the same, so \( \sin \theta = \frac{m \lambda}{d} = \frac{\lambda}{a} \Rightarrow m = \frac{d}{a} = \frac{0.150 \text{ mm}}{0.030 \text{ mm}} \)

or \( m = 5 \). That is, the fifth interference maximum on either side of the central maximum are missing, since the first diffraction minima fall on them, so \( m = 4 \) is the highest one observed within the central diffraction maximum. Therefore, the total number is \( 4 + 1 + 4 = 9 \).

\( \text{(on one side)} \) \( \text{centre} \) \( \text{(on the other side)} \)
4. (cont'd)

(a) (ii)

\[ I = I_{\text{max}} \cos^2 \left( \frac{\pi d \sin \theta}{\lambda} \right) \left[ \frac{\sin \left( \frac{\pi a}{\lambda} \sin \theta \right)}{\frac{\pi a}{\lambda} \sin \theta} \right]^2. \quad \text{Eqn (38.6)} \]

Since \( y = 2.20 \text{ mm}, \) \( \sin \theta = \tan \theta = \frac{y}{L} = \frac{2.20 \text{ mm}}{1800 \text{ mm}} = 0.001222. \)

Putting this into the equation above gives

\[ I = I_{\text{max}} \cos^2 \left( \frac{\pi (0.15 \times 10^{-3})(0.001222)}{550 \times 10^{-9}} \right) \left[ \frac{\sin \left( \frac{\pi (0.03 \times 10^{-3})(0.001222)}{550 \times 10^{-9}} \right)}{\frac{\pi (0.03 \times 10^{-3})(0.001222)}{550 \times 10^{-9}}} \right]^2, \]

or

\[ I = I_{\text{max}} \cos^2 \left( \frac{\pi}{3} \right) \left[ \frac{\sin \left( \frac{\pi}{15} \right)}{\frac{\pi}{15}} \right]^2 = I_{\text{max}} \left( \frac{1}{2} \right)^2 \left[ \frac{\sin \left( 0.20944 \right)}{0.20944} \right]^2, \]

\[ = I_{\text{max}} \left( \frac{1}{4} \right) (0.9927)^2 = I_{\text{max}} \left( \frac{1}{4} \right) (0.98546), \]

\[ \Rightarrow \quad I \approx 0.246 \, I_{\text{max}}. \]

(b) (i) Condition for destructive interference of reflected light:

\[ 2nt = m\lambda \Rightarrow t = \frac{m\lambda}{2n}. \]

The smallest \( t \) is for \( m = 1: \)

\[ t_{\text{mm}} = \frac{\lambda}{2n} = \frac{672 \text{ nm}}{2(1.6)} = 210 \text{ nm}. \]

(ii) Strong reflection \( \Rightarrow \) constructive interference:

\[ \lambda = \frac{2n \, t}{m + \frac{1}{2}} \Rightarrow \frac{2(1.6)(210 \text{ nm})}{m + \frac{1}{2}} = \boxed{448 \text{ nm}} \text{ (blue-violet)}. \]

Because this wavelength is strongly reflected, its transmission is concomitantly reduced.
(c) Use Malus's law: \( I = I_{\text{max}} \cos^2 \theta \):

\[
\begin{align*}
I_{\text{max}} \quad \text{(unpolarized)} \quad \rightarrow & \quad \begin{array}{c}
\text{Disc 1} \\
1
\end{array} \\
\frac{1}{2} I_{\text{max}} \quad \rightarrow & \quad \begin{array}{c}
\text{Disc 2} \\
\theta
\end{array} \\
\left(\frac{1}{2} I_{\text{max}}\right) \cos^2 \theta \quad \rightarrow & \quad \begin{array}{c}
\theta
\end{array} \\
\left(\frac{1}{2} I_{\text{max}} \cos^2 \theta\right) \cos^2 \theta
\end{align*}
\]

Final emergent intensity: \( \frac{1}{2} I_{\text{max}} \cos^4 \theta = 0.38 I_{\text{max}} \) (given)

\[\frac{1}{2} \cos^4 \theta = 0.38 \Rightarrow \cos^4 \theta = 0.76 \]

\[\cos \theta = (0.76)^{\frac{1}{4}} \approx 0.93369, \text{ or} \]

\[\theta = \cos^{-1}(0.93369) \Rightarrow \theta \approx 21^\circ.\]