We now consider the production of images in systems with more than one element. The procedure is very simple: the image of one element is the object for the next. We then just follow the light in this fashion until it emerges from the system. As an example consider the following system.

Lens 1 has a focal length of 20 cm, lens 2 has focal length 30 cm. An object 2 cm high is placed 15 cm to the left of the first lens. Where is the final image? How big is it? Is it erect or inverted? Is it real or virtual?

We begin with lens 1. Since it is a diverging lens we have:

\[ f = -20; \quad s = +15; \quad h = +2 \]

\[ s' = \frac{sf}{s-f} = \frac{15(-20)}{15-(-20)} = \frac{-300}{35} = -8.571 \text{ cm} \]

\[ h' = h \cdot \frac{s'}{s} = -2 \frac{-8.571}{15} = +1.143 \text{ cm} \]

The – sign means that the image is upstream from lens 1. Thus it is 38.571 cm upstream from lens 2. Lens 2 is a converging lens. Hence:

\[ f = +30; \quad s = +38.57; \quad h = +1.143 \]

\[ s' = \frac{38.57 \cdot 30}{38.57 - 30} = +135.0 \text{ cm} \]

\[ h' = -\frac{s'}{s} = -1.143 \frac{135}{38.57} = -4.00 \text{ cm} \]
The + sign means the image is downstream from the lens. Hence it is 110cm downstream from the mirror. This means that the object distance will be negative. Also the mirror is diverging and thus will have a negative focal length of $40/2 = 20$ cm. Putting these together we find:

\[ f = -20; \ s = -110; \ h = -4 \]

\[ s' = \frac{-110 \cdot (-20)}{-110 - (-20)} = -24.44 \text{ cm} \]

\[ h' = -(-4) \frac{24.44}{(-110)} = -0.8888 \text{ cm} \]

The - sign means the image is downstream from the mirror and hence right of the mirror. Now however the light is flowing from right to left. The image is 49.44 cm to the right of lens 2. Hence it is upstream and thus the object distance is positive. We then find:

\[ f = +30; \ s = 49.44; \ h = -0.8888 \]

\[ s' = \frac{49.44 \cdot 30}{49.44 - 30} = +76.30 \text{ cm} \]

\[ h' = +0.8888 \frac{76.30}{49.44} = +1.372 \text{ cm} \]

The + sign means the image is downstream from lens 2. Hence it is 36.30 cm downstream from lens 1. We thus get:

\[ f = -20; \ s = -36.30; \ h = +1.372 \]

\[ s' = \frac{-36.30 \cdot (-20)}{36.30 - (-20)} = +44.54 \text{ cm} \]

\[ h' = -1.372 \frac{(+44.54)}{-36.30} = +1.683 \text{ cm} \]

Thus the final image is 44.54 cm to the left of the lens 1, erect, 1.683 cm high, and real.

There is one situation where this procedure becomes a little complicated. That is if the image distance becomes infinite at one of the intermediate steps. To see how to handle this case consider the following arrangement.
**Lens**

\[ f = f_1; \ s = s_1; \ h = h \]

\[ s_1' = \frac{s_1 f_1}{s_1 - f_1} \]

\[ h_1' = -h \frac{f_1}{s_1 - f_1} \]

Now choose \( s_1 \) so that \( s_1' = L - f_2 \). Then we have:

**Mirror**

\[ f = f_2; \ s_2 = f_2; \ h_2 = -\frac{f_1 h}{s_1 - f_1} \]

\[ s_2' = \frac{f_2^2}{f_2 - f_2} = \infty \]

\[ h_2' = h \left( \frac{f_1}{s_1 - f_1} \right) \left( \frac{f_2^2}{f_2 - f_2} \right) = h \left( \frac{f_1}{s_1 - f_1} \right) \frac{f_2}{f_2 - f_2} \]

Now consider the lens again. For the moment we won’t worry about the singularity. We will just carry the symbols without evaluating them:

**Lens**

\[ s = L - s_2' \] (upstream is now right of lens); \( f = f_1; \ h = \frac{f_1}{s_1 - f_1} \frac{f_2}{f_2 - f_2} h \)
\[ s_3' = \left( \frac{L - s_2'}{L - s_2} - f_1 \right) f_1 = \left( L - \frac{f_2^2}{f_2 - f_2} \right) f_1 = \frac{\left[ L(f_2 - f_2) - f_2^2 \right] f_1}{(f_2 - f_2)} = \frac{-f_2^2 f_1}{-f_2^2} = f_1 \]

Of course it does, since light coming from infinity is focused at the focal point. Remember that that was the definition of the focal point. Now what about the size of the final image? We have:

\[ h_3' = -h \frac{f_1}{s_1 - f_1} \left( \frac{f_2}{f_2 - f_2} \right) \frac{f_1}{L - \frac{f_2^2}{f_2 - f_2}} = -h \left( \frac{f_1}{s_1 - f_1} \right) \frac{f_1 f_2}{(f_2 - f_2) L - f_2^2} = +h \frac{f_1}{s_1 - f_1} \frac{f_1}{f_2} \]

But

\[ s_1 = \frac{s_1' f_1}{s_1' - f_1} = \frac{(L - f_2) f_1}{L - f_2 - f_1} \]

Hence

\[ h_3' = h \frac{f_1^2}{f_2 \left[ \frac{(L - f_2) f_1}{L - f_2 - f_1} - f_1 \right]} = \frac{h (L - f_1 - f_2)}{f_2} \]

In this manner we can handle any singularity that arises.