CONDUCTORS AND INSULATORS

We define a conductor as a material in which charges are free to move over macroscopic distances—i.e., they can leave their nuclei and move around the material. An insulator is anything else. In an insulator the charge distribution in an atom may change, but the charges do not leave their nuclei.

When we consider electrostatics, the case where charges are not moving, this definition has several immediate consequences.

1. There can be no electric field inside a conductor. If there were, it would exert a force on the charges causing them to move. They would adjust until there was no field.

2. There can be no net charge at any point inside a conductor. Gauss’s Law would give

\[ \oint \vec{E} \cdot d\vec{A} = 0 = 4\pi kQ \rightarrow Q = 0 \]

3. The electric field at the surface of the conductor must be perpendicular (\(\perp\)) to the surface. If there were a component parallel to the surface it would cause charges to move along the surface until there was none.

There are other consequences we will investigate later.

Symmetries

Gauss’s Law is always true, but in integral form it is generally only useful if we can get \(\vec{E}\) outside the surface integral. This generally requires a special symmetry. We will examine three such symmetries.

Spherical Symmetry

Consider the following arrangement.
It consists of two hollow concentric conducting shells as shown. A charge $Q_0$ is placed at the center, $Q_1$ on the inner shell, and $Q_2$ on the outer shell. The rest of the space is a vacuum. Find the electric field and charge density everywhere.

$$ r < R_1 $$

Draw an imaginary sphere of radius $r$ as shown.

Because of the spherical symmetry we know that the magnitude of $\vec{E}$ is the same everywhere on the sphere of radius $r$, and also that it is radial (points in $\hat{r}$ direction). Thus.

$$ \oint \vec{E} \cdot d\vec{A} = \oint E dA \cos(0) = E \oint dA = E 4\pi r^2 = 4\pi kQ_0 \rightarrow \vec{E} = \frac{kQ_0}{r^2} \hat{r} $$

$$ R_1 \geq r \leq R_2 $$

Again, draw an imaginary sphere in this region.

Since the sphere is inside the conductor we know $\vec{E} = 0$. But then Gauss’s Law tells us the total charge inside the sphere is 0. This consists of $Q_0$ and $Q_{in}$ where $Q_{in}$ is the charge on the inner
surface of the conductor. Since there can be no charge in the conductor, and the total charge on the conductor is \( Q_1 \), we must have

\[
Q_{1\text{out}} = Q_0 + Q_1
\]

\[
Q_{1\text{in}} = -Q_0
\]

\[
\vec{E} = 0
\]

\( R_2 < r < R_3 \)

This is the same as the 1st region with \( Q_0 \) replaced by \( Q_0 + Q_1 \)

\[
\vec{E} = \frac{k(Q_0 + Q_2)}{r^2} \hat{r}
\]

\( R_3 \leq r \leq R_4 \)

This is the same as region 2

\[
Q_{2\text{out}} = Q_0 + Q_1 + Q_2
\]

\[
Q_{2\text{in}} = -Q_0 - Q_1
\]

\[
\vec{E} = 0
\]

\( r > R_4 \)

This is the same as regions 1 and 3

\[
\vec{E} = \frac{k(Q_0 + Q_1 + Q_2)}{r^2} \hat{r}
\]

We see that the fields are either 0 or the same as if all charges were at the center.

Cylindrical Symmetry

Next we consider a conducting cylinder of radius \( R \) and length \( L \) carrying a charge \( Q_0 \).
Because of the cylindrical symmetry we know that the field magnitude must be independent of θ (the angle about the axis of the cylinder). Further we know that $\mathbf{E}$ must be $\perp$ to the surface of the cylinder. We do not know that it has to stay that way. However, at center it must be $\perp$ by symmetry.

To see what happens at other points we note that if we go far away from the cylinder compared to R or L, it would appear as a point charge. Hence the field will appear to be radial at long distances.

Since it must start $\perp$ to the cylinder it must look like
Thus if we stay close to the cylinder ($r \ll L$) the lines will be $\perp$ to the cylinder (directed along the radius of the cylinder). If we use cylindrical coordinates with $\hat{z}$ along the axis this becomes

$$\vec{E} = E(\rho, z) \hat{\rho}$$

The remaining question is the $z$ dependence. We expect the charge density to be greater near the ends than at the center because like charges repel (at the right hand end there is no charge pushing back, to the left, hence charge will tend to accumulate there). However, if we are near the middle we can’t see either end and hence there should be no difference in charge density if we look toward either end. This means it must be uniform. Thus if we stay away from the ends we expect uniform charge density

$$\rho \sim \frac{Q}{\pi R^2 L}$$

We will thus approximate the situation as one in which $E$ does not depend on $z$. Then draw an imaginary cylinder of radius $r$ as shown.
Then

\[ \int \vec{E} \cdot d\vec{A} = \int E \, dA \cos \theta = E \, 2\pi r \ell = 4\pi kQ_{\text{in}} \frac{\ell}{L} \]

\[ \therefore \vec{E} = \frac{2kQ_0}{Lr} \hat{\rho} \]

Note that the field falls off as \( 1/r \) rather than \( 1/r^2 \). This is because the lines spread out in one dimension rather than two. This same phenomenon has interesting implications in modern string theories of more than 3 dimensions.

Next consider the analogous problem to the concentric conducting spheres.

\[ \rho < R_1 \]

\[ \vec{E} = \frac{2kQ_1}{Lr} \hat{\rho} \]

\[ R_1 \leq \rho \leq R_2 \]

\[ Q_{\text{in}} = -Q_0 \]

\[ Q_{\text{out}} = Q_0 + Q_1 \]

\[ \vec{E} = 0 \]

\[ R_2 \leq \rho \leq R_3 \]
Planar Symmetry

Finally, we consider the case of a plane carrying a charge $Q_0$. We do this just as we did for the cylinders. The charge will tend to cluster near the edge, but if we stay away from them it will be uniform. The field will start $\perp$ to the surface and then bend toward the edges when the distance from the sheet becomes comparable to $L$. Hence, away from the edges and close to the sheet we draw a rectangular box
By symmetry the field is the same above and below the sheet. Thus

\[ \iiint \vec{E} \cdot d\vec{A} = \int \int \vec{E} \cdot d\vec{A} \cos \theta = E \cdot dA \cdot 2\pi a = 4\pi kQ_0 \frac{a}{A} \]

\[ \therefore \vec{E} = 2\pi k \frac{Q}{A} \hat{n} \equiv 2\pi k \sigma \hat{n} \]

where \( \sigma \) is the surface charge density and \( \hat{n} \) is the normal to the surface.

Note that this time the field does not fall off with distance—obviously because the lines don’t spread!!