You may use one sheet of notes and formulas, but you must not collaborate with any other person. Do all four problems, showing your method and working clearly (a correct answer alone is not necessarily sufficient). Be sure to include correct units in your answers where appropriate. The number of marks for each part is given in square brackets, [ ], to the right of the question.

Stefan-Boltzmann constant: \( \sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{K}^4 \). Wien displacement law constant: \( h\lambda = 1240 \text{ eV-nm} \). Boltzmann constant: \( k = 8.617 \times 10^{-5} \text{ eV/K} \).

1. A spherical blackbody has radius 12.0 cm and is at Kelvin temperature \( T \). At a distance of 100 m from the blackbody, the intensity of the electromagnetic radiation emitted by it (including all wavelengths) is measured to be 2.71 W/m\(^2\).

(a) Calculate the value of \( T \). [10]

(b) Calculate the value of \( \lambda_m \), the wavelength corresponding to the peak of the spectral radiance curve for this blackbody. [2]

(c) If the temperature of the blackbody is increased so as to double its total power output, what will be the new value of \( \lambda_m \)? [6]

(d) If the spectral radiance at wavelength \( \lambda_m \) is \( R_m \), what is the spectral radiance at wavelength \( \lambda_m/2 \)? (Use the Planck radiation law, and give your answer as a decimal number, correct to two decimal places, times \( R_m \).) [10]

(a) From Stefan's law, \( R = \sigma T^4 \) is the power radiated per surface area of the blackbody, so power \( P = 4\pi r^2 \sigma T^4 \), where \( r \) is the radius of the blackbody. The intensity \( I \) at distance \( R \) from the blackbody is \( I = \frac{P}{4\pi R^2} = \frac{4\pi r^2 \sigma T^4}{4\pi R^2} = \frac{r^2 \sigma T^4}{R^2} \)

\[ \Rightarrow T = \sqrt[\frac{4\pi R^2}{r^2 \sigma}] I = \sqrt[\frac{(100)^2}{(0.12)^2}(2.71)} \approx \sqrt[5.67 \times 10^{-8}] \approx \sqrt[3.319 \times 10^{13}] \approx 2400 \text{ K} \]

\[ \Rightarrow T \approx 2400 \text{ K} \]

(b) \( \lambda_m T = \text{constant} = 2.898 \times 10^6 \text{ nm.K} \)

\[ \Rightarrow \lambda_m = \frac{2.898 \times 10^6 \text{ nm-K}}{2400 \text{ K}} \Rightarrow \lambda_m \approx 1210 \text{ nm} \]
(c) If the power is doubled, then the new temperature $T'$ is related to the original temperature $T$ by $\sigma(T')^4 = 2\sigma T^4$, or $T' = \sqrt{2}T$. Using this in the Wien displacement law, we see that

$$\lambda_m' = \frac{2.898 \times 10^6 \text{nm-K}}{T'} = \frac{2.898 \times 10^6 \text{nm-K}}{\sqrt{2}T}$$

$$\Rightarrow \lambda_m' = \frac{\lambda_m}{\sqrt{2}} \approx 1020 \text{nm}.$$ 

(d) Using $R(\lambda) = \frac{1}{4}c\tau(\lambda)$ and Planck's radiation law for $\tau(\lambda)$, we have

$$\frac{R(\frac{\lambda_m}{2})}{R(\lambda_m)} = \frac{8\pi hc (\frac{\lambda_m}{2})^{-5} / [e^{\frac{hc}{k\lambda_mT}} - 1]}{8\pi hc (\lambda_m)^{-5} / [e^{\frac{hc}{k\lambda_mT}} - 1]} = 32 \left[ e^{\frac{hc}{k\lambda_mT}} - 1 \right]$$

Now $\lambda_mT = 2.898 \times 10^6 \text{nm-K}$, $hc = 1240 \text{nm-ev}$, and $k = 8.617 \times 10^{-5} \text{ev/K}$, so

$$\frac{R(\frac{\lambda_m}{2})}{R_m} = 32 \left[ e^{\frac{1240}{(8.617 \times 10^{-5})(2.898 \times 10^6)}} - 1 \right] \approx 32 \left[ e^{\frac{2.480}{9.9311}} - 1 \right] \approx 32 \left[ e^{0.96555} - 1 \right]$$

$$\Rightarrow \frac{R(\frac{\lambda_m}{2})}{R_m} \approx 32 \left[ \frac{14.387}{20.559} \right] \approx 0.2216 \Rightarrow R(\frac{\lambda_m}{2}) \approx 0.22R_m.$$
2. Ultraviolet light of wavelength 200 nm is shone on to a certain clean metal surface, and the stopping potential for photoemission of electrons is measured. When the incident wavelength is changed to 139 nm, the stopping potential is now found to be twice its previous value. Calculate:

(a) the work function for this metal;  [12]  
(b) the stopping potential if light of wavelength 180 nm is used;  [8]  
(c) the threshold wavelength for photoemission from this metal.  [4]

(a) Suppose that $V_0$ is the stopping potential for $\lambda = 200$ nm:

$$e V_0 = hf - \phi = \frac{hc}{\lambda} - \phi = \frac{1240}{200} - \phi \Rightarrow e V_0 = 6.2 \text{ eV} - \phi. \quad (1)$$

The stopping potential for $\lambda = 139$ nm is $2V_0$, so

$$e (2V_0) = \frac{1240}{139} - \phi \Rightarrow (2eV_0 = 8.92 \text{ eV} - \phi). \quad (2)$$

Multiplying equation (1) by 2, these equations become

$$\begin{cases} 2eV_0 = 12.8 \text{ eV} - 2\phi \\ 2eV_0 = 8.92 \text{ eV} - \phi \end{cases}$$

Subtracting these gives $0 = 3.88 \text{ eV} - \phi \Rightarrow \phi = 3.48 \text{ eV}$.

(b) We have $eV = \frac{hc}{\lambda} - \phi = \frac{1240}{180} - 3.48 \approx 6.89 - 3.48 \text{ eV}$

$$\Rightarrow eV = 3.41 \text{ eV} \Rightarrow V = 3.41 \text{ V}.$$

(c) \[ \frac{hc}{\lambda_t} = \phi \Rightarrow \lambda_t = \frac{hc}{\phi} = \frac{1240 \text{ eV nm}}{3.48 \text{ eV}} \Rightarrow \lambda_t \approx 356 \text{ nm}. \]
3.

In a Compton scattering experiment, an x-ray photon of initial energy \( E_1 = 100 \text{ keV} \) and wavelength \( \lambda_1 \) travelling along the x-axis in the positive direction is incident on a free electron at rest. The photon is scattered in the xy-plane at an angle of \( 120^\circ \) with the positive x-axis, as shown in the sketch above. The energy of the scattered photon is \( E_2 \) and its wavelength is \( \lambda_2 \).

(You may assume the usual Compton scattering formula, and recall that the rest energy of an electron is 511 keV. Giving wavelengths in nm, energies in keV, and momenta in keV/c, calculate the values of:

(a) \( \lambda_1 \) [2]

(b) \( \lambda_2 \) [6]

(c) \( E_2 \) [2]

(d) \( K \), the kinetic energy of the scattered electron; [2]

(e) the x- and y-components of the momentum of the scattered photon; [8]

(f) the x- and y-components of the momentum of the scattered electron; [4]

(g) the scattering angle, \( \phi \), of the electron. [2]

(a) \( \lambda_1 = \frac{hc}{E_1} \approx \frac{1240 \text{ eV} \cdot \text{nm}}{100000 \text{ eV}} = 0.0124 \text{ nm} \)

(b) \( \lambda_2 - \lambda_1 = 0.00243 \text{ nm} (1 - \cos \Theta) \Rightarrow \lambda_2 = \lambda_1 + 0.00243 \text{ nm} (1 - \cos \Theta) \)

\( \text{Compton scattering formula} \)

or \( \lambda_2 \approx 0.0124 \text{ nm} + 0.00243 \text{ nm} (1 - \cos 120^\circ) \Rightarrow \lambda_2 \approx 0.016045 \text{ nm} \)

(c) \( E_2 = \frac{hc}{\lambda_2} \approx 77.3 \text{ keV} \)

(d) \( K = E_1 - E_2 = 100 \text{ keV} - 77.3 \text{ keV} \Rightarrow K \approx 22.7 \text{ keV} \).
(e) The momentum of the photon (in magnitude) is \( \frac{E_\gamma}{c} = 77.3 \text{ keV} \).

Its \textit{x-component} is \( \left( \frac{77.3 \text{ keV}}{c} \right) \cos 120^\circ = -38.6 \text{ keV} \).

Its \textit{y-component} is \( \left( \frac{77.3 \text{ keV}}{c} \right) \sin 120^\circ = +66.9 \text{ keV} \).

(f) The original photon had momentum \( \frac{100 \text{ keV}}{c} \) in the \textit{x}-direction.

Therefore, if \( p_x \) is the \textit{x-component} of the momentum of the scattered electron, momentum conservation requires that

\[
p_x + \frac{-38.6 \text{ keV}}{c} = \frac{100 \text{ keV}}{c} \Rightarrow p_x = \frac{138.6 \text{ keV}}{c}
\]

By similar reasoning, \( p_y = \frac{-66.9 \text{ keV}}{c} \).

(g) \[ \tan \phi = \frac{|p_y|}{|p_x|} \approx \frac{66.9}{138.6} \approx 0.483 \Rightarrow \phi \approx \tan^{-1}(0.483), \]

so \( \phi \approx 25.8^\circ \).
4. Suppose that a muon (207 times as massive as an electron and carrying charge \(-e\)) and a pion (273 times as massive as an electron, with charge +e) orbit their common centre of mass to form a muon-pion ‘atom’. Use the Bohr model to answer the questions below concerning the Balmer series (transitions from \(n_i > 2\) to \(n_f = 2\)) of this atom.

(a) Calculate the numerical value of the Rydberg constant, \(R\), for this atom, giving your answer correct to four significant figures. (Suggestion: Do this by first expressing \(R\) as a fraction times \(R_\infty\), whose value is \(1.097373 \times 10^7 \text{ m}^{-1}\).) \([10]\)

(b) Find both the value of \(n_i\) and the wavelength of the emitted photon (correct to three significant figures) for:

- (i) the longest wavelength Balmer line; \([6]\)
- (ii) the shortest wavelength Balmer line. \([6]\)

(a) Since the masses of the muon and the pion are comparable, we must use the reduced mass \(\mu\) to calculate \(R\). If \(m\) is the mass of an electron, we have \(\mu = \frac{(207m)(273m)}{207m + 273m} = \frac{56511}{480} \text{ m}^2\)

\[\Rightarrow \mu = \frac{18837}{160} \text{ m}.\] Therefore,

\[R = \frac{\mu k^2 e^4}{4\pi\hbar^2 c} = \frac{\mu}{m} \left(\frac{mk^2 e^4}{4\pi\hbar^2 c}\right) \Rightarrow R = \frac{\mu}{m} R_\infty = \frac{18837}{160} R_\infty,\]

or \(R \approx 117.7 R_\infty\), or

\[R = 117.7 \times (1.097373 \times 10^7 \text{ m}^{-1}) \Rightarrow R \approx 1.292 \times 10^9 \text{ m}^{-1}.\]

(b) In this case \(Z = 1\) (since the charge on the pion is +e), so \(\frac{1}{\lambda} = R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)\). Since \(n_f = 2\) (Balmer series), we have

\[\lambda = \frac{1}{R \left(\frac{1}{4} - \frac{1}{n_i^2}\right)}.\] (P.T.O.)
(b) (cont'd)

(i) The longest wavelength line is the least energetic one of the series, so \( n_i = 3 \), and

\[
\lambda_{\text{max}} = \frac{1}{R \left( \frac{1}{4} - \frac{1}{9} \right)} = \frac{1}{(1.292 \times 10^9 \, \text{m}^{-1}) \left( \frac{5}{36} \right)} \Rightarrow \lambda_{\text{max}} \approx 5.57 \, \text{nm}
\]

(ii) The shortest wavelength line is the most energetic one in the series, so \( n_i = \infty \), and

\[
\lambda_{\text{min}} = \frac{1}{R \left( \frac{1}{4} - \frac{1}{\infty} \right)} = \frac{1}{(1.292 \times 10^9 \, \text{m}^{-1}) \left( \frac{1}{4} \right)} \Rightarrow \lambda_{\text{min}} \approx 3.10 \, \text{nm}
\]