Physics 3740 HW Nº 2 Ch. 2: 3, 11, 14, 18, 21, 26, 27, 29, 45, 46, 48

3

2-3. An electron of rest energy $mc^2 = 0.511$ MeV moves with respect to the laboratory at speed $u = 0.6c$. Find (*a*) γ, (*b*) p in units of MeV/c, (*c*) E, and (*d*) E_k.

2-3. (a) $\gamma = \dfrac{1}{\sqrt{1-u^2/c^2}} = \dfrac{1}{\sqrt{1-(0.6c)^2/c^2}} = \dfrac{1}{0.8} = 1.25$

(b) $p = \gamma m u = \gamma(mc^2)(u/c)/c = 1.25(0.511MeV)(0.6)/c = 0.383\,MeV/c$

(c) $E = \gamma mc^2 = 1.25(0.511MeV) = 0.639\,MeV$

(d) $E_k = (\gamma - 1)mc^2 = 0.25(0.511MeV) = 0.128\,MeV$

11

2-11. An electron with rest energy of 0.511 MeV moves with speed $u = 0.2c$. Find its total energy, kinetic energy, and momentum.

2-11. $E = \gamma m c^2$ (Equation 2-10)

where $\gamma = 1/\sqrt{1-u^2/c^2} = 1/\sqrt{1-(0.2)^2} = 1.021 \longrightarrow (1.020621)$

$E = (1.021)(0.511MeV) = 0.522\,MeV \longrightarrow (0.521537)$

$E_k = \gamma mc^2 - mc^2 = mc^2(\gamma - 1)$ (Equation 2-9)

$= (0.511MeV)(1.021-1) = 0.011\,MeV$

$E^2 = (pc)^2 + (mc^2)^2$ (Equation 2-31)

$p^2 = \dfrac{1}{c^2}\left(E^2 - (mc^2)^2\right) = \dfrac{1}{c^2}\left[(0.522MeV)^2 - (0.511MeV)^2\right] \longrightarrow \dfrac{0.01088}{c^2}(MeV)^2$

$\underbrace{= 0.0114/c^2}_{0.01088\,\frac{MeV^2}{c^2}\text{ is closer.}} \Rightarrow \underbrace{p = 0.107\,MeV/c}_{0.104\,\frac{MeV}{c}\text{ is closer}}$

An easier way to get p: Since $p = \gamma m u$ and $E = \gamma mc^2$,

(see Eq 2-41) $\frac{p}{E} = \frac{u}{c^2}$, so $p = \frac{Eu}{c^2} \simeq \frac{(0.521537\ MeV)(0.2c)}{c^2}$

$\Rightarrow \boxed{p \simeq 0.104\ \frac{MeV}{c}}$.

(14)

2-14. An electron in a hydrogen atom has a speed about the proton of 2.2×10^6 m/s. (*a*) By what percent do the relativistic and Newtonian values of E_k differ? (*b*) By what percent do the momentum values differ?

2-14. $u = 2.2\times 10^6 m/s$ and $\gamma = 1/\sqrt{1-u^2/c^2}$

(a) $E_k(\text{rel}) = mc^2(\gamma-1) = 0.5110MeV(\gamma-1) = 0.5110MeV\left(1/\sqrt{1-u^2/c^2}-1\right)$

$\simeq 0.5110\ MeV\ (1.00002689 - 1) = 0.5110\ MeV\ (0.00002689)$

$\approx 1.374079 \times 10^{-5}\ MeV$

$E_k(\text{classical}) = \frac{1}{2}mu^2 = \frac{1}{2}mc^2(u^2/c^2) = (0.5110MeV/2)(2.2\times10^6/c)^2$

$\simeq 1.374022 \times 10^{-5}\ MeV$

$\%\ \text{difference} = \dfrac{0.000057\times10^{-5}}{1.374079\times10^{-5}} \times 100 \approx \boxed{0.00415\ \%}$

$p\,(\text{relativistic}) = \frac{1}{c}\sqrt{E^2-(mc^2)^2} = \frac{1}{c}\sqrt{(\gamma mc^2)^2-(mc^2)^2}$

$= mc\sqrt{\gamma^2-1} = \dfrac{mc^2}{c}\left[\left(1/\sqrt{1-(2.2\times10^6/3.0\times10^8)^2}\right)^2-1\right]^{1/2}$

$= 0.5110MeV/c(7.33353\times10^{-3}) \simeq 3.747434\times10^{-3}\ \frac{MeV}{c}$

$p(\text{classical}) = mu = \dfrac{mc^2}{c}\left(\dfrac{u}{c}\right) = (0.5110MeV/c)(2.2\times10^6/3.0\times10^8)$

$= 3.747333\times10^{-3}\ MeV$

$\%\ \text{difference} = \dfrac{1.01\times10^{-7}}{3.747434\times10^{-3}} \times 100 = \boxed{0.0027\ \%}$

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(18)

2-18. The energy released when sodium and chlorine combine to form NaCl is 4.2 eV. (*a*) What is the increase in mass (in unified mass units) when a molecule of NaCl is dissociated into an atom of Na and an atom of Cl? (*b*) What percentage of error is made in neglecting this mass difference? (The mass of Na is about 23 u, and that of Cl is about 35.5 u.)

2-18. (a) $\Delta m = \dfrac{\Delta E}{c^2} = \dfrac{\Delta E\,u}{uc^2} = \dfrac{4.2eV}{931.5\times10^6 eV}u = 4.5\times10^{-9}\,u$

(b) $\text{error} = \dfrac{\Delta m}{m(Na)+m(Cl)} = \dfrac{4.5\times10^{-9}u}{23u+35.5u} = 7.7\times10^{-11} = 7.7\times10^{-9}\,\%$

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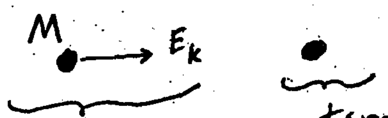
(21) **2-21.** When a beam of high-energy protons collides with protons at rest in the laboratory (e.g., in a container of water or liquid hydrogen), neutral pions (π^0) are produced by the reaction $p + p \rightarrow p + p + \pi^0$. Compute the threshold energy of the protons in the beam for this reaction to occur. (See Table 2-1 and Example 2-13).

Let M denote the mass of a proton and m the mass of a π^0. Pictorially, the situation is as follows:

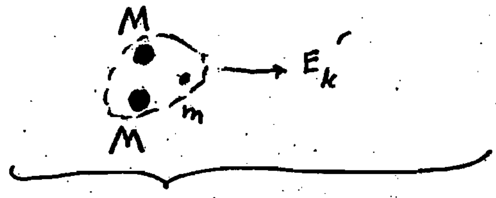
before collision (i ... initial)



incoming proton of kinetic energy E_k

target proton at rest

after collision (f ... final)



two protons and one π^0 moving off together (all with same velocity; total K.E. E_k')

In general, E_k (the kinetic energy of an incoming proton that is necessary to result in the creation of a π^0) will be a <u>minimum</u> (called the 'threshold energy') when the final three particles (2 protons and the π^0) <u>do not move apart from each other after the collision</u>. If they did, we should have to supply the incoming particle with additional kinetic energy. Remembering that $E = E_k + E_0 = E_k + mc^2$ and that $p = \frac{1}{c}\sqrt{E^2 - (mc^2)^2} = \frac{1}{c}\sqrt{(mc^2 + E_k)^2 - (mc^2)^2}$,

the principles of conservation of momenergy applied to the figure above give us:

<u>conservation of energy</u>: $E_f = E_i$, or $2Mc^2 + mc^2 + E_k' = Mc^2 + E_k + Mc^2$,

or $$E_k = E_k' + mc^2 \quad (1)$$

(P.T.O.)

(21) (cont'd)

<u>conservation of momentum</u>: $p_f = p_i$, or

(the stationary target proton has 0 momentum)

$$\frac{1}{c}\sqrt{(2Mc^2 + \underbrace{mc^2 + E_k'}_{=E_k,\text{ from (1)}})^2 - (2Mc^2 + mc^2)^2} = \frac{1}{c}\sqrt{(Mc^2+E_k)^2 - (Mc^2)^2}$$

$$\Rightarrow (2Mc^2 + E_k)^2 - (2Mc^2 + mc^2)^2 = (Mc^2 + E_k)^2 - (Mc^2)^2, \text{ or}$$

$$\cancel{4(Mc^2)^2} + 4Mc^2E_k + \cancel{E_k^2} - \cancel{4(Mc^2)^2} - 4(Mc^2)(mc^2) - (mc^2)^2 = \cancel{(Mc^2)^2} + 2(Mc^2)E_k + \cancel{E_k^2} - \cancel{(Mc^2)^2}$$

$$\Rightarrow 2(Mc^2)E_k = 4(Mc^2)(mc^2) + (mc^2)^2,$$

so
$$E_k = \frac{4(Mc^2)(mc^2) + (mc^2)^2}{2(Mc^2)}$$

Using the values given in Table 2-1 (p. 84), we have

$Mc^2 = 938.28$ MeV (rest energy of a proton) and

$mc^2 = 135$ MeV (rest energy of a neutral pion). Putting these numbers into the formula derived on the previous page, we have for the threshold energy:

$$E_k = \frac{4(938.28)(135) + (135)^2}{2(938.28)} \approx \boxed{279.7 \text{ MeV}} \approx \boxed{280 \text{ MeV}}.$$

An Example Problem (not assigned):

The K^0 particle decays according to the equation $K^0 \to \pi^+ + \pi^-$. If a particular K^0 decays while at rest in the laboratory, what are the kinetic energies of each of the two pions? (The rest mass of the K^0 is 497.7 MeV/c^2.)

Conservation of momentum requires the pions to be emitted with equal and opposite momenta. Therefore, since their masses are equal, their kinetic energies will also be equal. The kinetic energy of each pion is given by

$$\frac{\text{total KE}}{2} = \frac{(\text{rest energy of } K^0) - (\text{total rest energies of two pions})}{2}$$

$$= \frac{497.7\text{ MeV} - 2(139.6\text{ MeV})}{2} = \boxed{109.25\text{ MeV}}.$$

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(26) **2-26.** Show that Equation 2-31 can be written $E = mc^2(1 + p^2/m^2c^2)^{1/2}$, and use the binomial expansion to show that, when pc is much less than mc^2, $E \approx mc^2 + p^2/2m$.

$$E^2 = (mc^2)^2 + (pc)^2 = (mc^2)^2\left[1 + \left(\frac{pc}{mc^2}\right)^2\right] = (mc^2)^2\left[1 + \left(\frac{p}{mc}\right)^2\right]$$

$\Rightarrow E = mc^2\left[1 + \left(\frac{p}{mc}\right)^2\right]^{\frac{1}{2}}$. Using the binomial expansion,

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)}{3!}x^3 + \dots \approx 1 + nx \text{ if } x \ll 1,$$

with $x = \left(\frac{p}{mc}\right)^2 = \left(\frac{pc}{mc^2}\right)^2 << 1$, we have $E \approx mc^2\left[1 + \frac{1}{2}\left(\frac{p}{mc}\right)^2\right]$, or

$E \approx mc^2 + \frac{mc^2}{2}\frac{p^2}{m^2c^2} \Rightarrow \boxed{E \approx mc^2 + \frac{p^2}{2m}}$. Q.E.D. ($\frac{p^2}{2m}$ is the Newtonian expression for kinetic energy.)

27

2-27. An electron of rest energy 0.511 MeV has a total energy of 5 MeV. (*a*) Find its momentum in units of MeV/c. (*b*) Find *u/c*.

2-27. $E^2 = (pc)^2 + (mc^2)^2$ (Equation 2-31)

(a) $(pc)^2 = E^2 - (mc^2)^2 = (5MeV)^2 - (0.511MeV)^2 = 24.74$

Or $p = \sqrt{24.74}/c = 4.97 MeV/c$

(b) $E = \gamma mc^2 \Rightarrow \gamma = E/mc^2 = 1/\sqrt{1-u^2/c^2} \Rightarrow 1-u^2/c^2 = (mc^2/E)^2$

$u/c = [1-(mc^2/E)^2]^{1/2} = [1-(0.511/5.0)^2]^{1/2} = 0.995.$

Alternatively, use Eqn (2-34), p. 87, to get $\frac{u}{c}$ from

$$\frac{u}{c} = \frac{pc}{E} \underset{[\text{from (a)}]}{\simeq} \frac{\sqrt{24.74}}{5} \simeq 0.995.$$

29

2-29. What is the speed of a particle observed to have momentum 500 MeV/c and energy 1746 MeV? What is the particle's mass (in MeV/c^2)?

(a) From Eqn 2-34, p. 87, $u = \frac{pc^2}{E} = \frac{\left(500\,\frac{MeV}{c}\right)c^2}{1746\ MeV} \Rightarrow \boxed{u \simeq 0.286\,c}$.

(b) $E^2 = (mc^2)^2 + (pc)^2 \Rightarrow m = \sqrt{\left(\frac{E}{c^2}\right)^2 - \left(\frac{p}{c}\right)^2}$, so here

$$m = \sqrt{\left(1746\,\frac{MeV}{c^2}\right)^2 - \left(500\,\frac{MeV}{c^2}\right)^2} \Rightarrow \boxed{m \simeq 1673\ \frac{MeV}{c^2}}.$$

45

2-45 Show that the creation of an electron-positron pair (or any particle-antiparticle pair, for that matter) by a single photon is not possible in isolation, i.e., that additional mass (or radiation) must be present. (*Hint:* Use the conservation laws.)

The easiest way to show this is to analyse the process in the centre-of-momentum frame of the electron-positron pair. In this frame the total momentum after the pair production is <u>zero</u>, since the two particles have equal and opposite momenta. By conservation of momentum, the momentum must have also been zero before the pair production. But there is no frame in which the momentum of a single photon is zero. Therefore, such a process is impossible.

(46) 2-46 With inertial systems S and S' arranged with their corresponding axes parallel and S' moving in the $+x$ direction, it was apparent that the Lorentz transformation for y and z would be $y' = y$ and $z' = z$. The transformation for the y and z components of the momentum are not so apparent, however. Show that, as stated in Equations 2-16 and 2-17, $p'_y = p_y$ and $p'_z = p_z$.

$$2\text{-}46 \quad p'_y = \gamma' m u'_y = \overbrace{\left[\frac{\gamma\left(1 - vu_x/c^2\right)}{\sqrt{1-u^2/c^2}}\right]}^{\substack{\gamma' \\ \text{(see eq. 2-13 on p. 73)}}} \times m \times \overbrace{\left[\frac{u_y}{\gamma\left(1-u_x v/c^2\right)}\right]}^{\substack{u'_y \\ \text{(see p. 27)}}}$$

Canceling γ and $\left(1 - vu_x/c^2\right)$, gives: $p'_y = \dfrac{mu_y}{\sqrt{1-u^2/c^2}} = p_y = \gamma m u_y$.

In an exactly equivalent way, $p'_z = p_z$.

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(48) 2-48. An antiproton $\bar{p}$ has the same rest energy as a proton. It is created in the reaction $p + p \rightarrow p + p + p + \bar{p}$. In an experiment, protons at rest in the laboratory are bombarded with protons of kinetic energy E_k, which must be great enough so that kinetic energy equal to $2mc^2$ can be converted into the rest energy of the two particles. In the frame of the laboratory, the total kinetic energy cannot be converted into rest energy because of conservation of momentum. However, in the zero-momentum reference frame in which the two initial protons are moving toward each other with equal speed u, the total kinetic energy can be converted into rest energy. (*a*) Find the speed of each proton u such that the total kinetic energy in the zero-momentum frame is $2mc^2$. (*b*) Transform to the laboratory's frame in which one proton is at rest, and find the speed u' of the other proton. (*c*) Show that the kinetic energy of the moving proton in the laboratory's frame is $E_k = 6mc^2$.

2-48 (a) Each proton has $E_k = m_p c^2(\gamma - 1)$ and since we want $E_k = m_p c^2$, then $\gamma = 2$ and $u = 0.866c$.

(b) In the lab frame S:

$u'_x = \dfrac{u_x - v}{1 - u_x v/c^2}$ where $u = v$ and $u_x = -u$ yields:

$$u'_x = \frac{-2u}{1+u^2/c^2} = \frac{-2(0.866c)}{1+(0.866)^2} = -0.990c$$

(c) For $u'_x = -0.990c$, $\gamma = 1/\sqrt{1-(0.99)^2} = 7.0$ and the necessary kinetic energy in the lab frame S is : $E_k = m_p c^2(\gamma - 1) = m_p c^2(7-1) = 6m_p c^2$