A. Single-layer graphene is a two-dimensional gapless semiconductor with mirror symmetry of conduction and valence bands (Fig. 1). The dispersion law, which is the dependence of electron energy, ε, on momentum, $\vec{p}$, is linear, $\varepsilon = |\vec{p}|\,v$. At zero temperature, T = 0, the conduction band is empty, while the valence band is completely occupied.

1. With accuracy of a numeric coefficient find the concentrations of electrons and holes at finite temperature T.
2. Find the average electron energy and the specific heat.

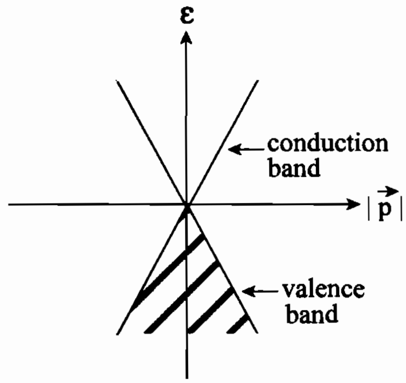
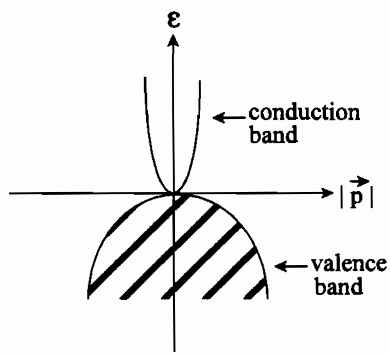


Fig. 1

B. In typical three dimensional gapless semiconductor the dispersion laws for electrons and holes are quadratic: $\varepsilon_e = \frac{p^2}{2me}$, $\varepsilon_h = -\frac{p^2}{2m_h}$. The hole mass is much bigger than the electron mass $m_h >> m_e$. Using this fact, perform the same calculations 1 and 2 as in part A.



Solution Part I

Density of electron states

$$g_e(\varepsilon) = 2\int \frac{d^2p}{(2\pi\hbar)^2}\,\delta(\varepsilon - V|p|) = \frac{\varepsilon}{\pi V^2\hbar^2}$$

Concentrations of electrons and holes are equal. Due to the symmetry of conduction and valence bands the Fermi level is zero, $\mu = 0$ at any T

Electron concentration $n_e = \int_0^\infty d\varepsilon \frac{g_e(\varepsilon)}{e^{\frac{\varepsilon}{T}}+1} = \frac{T^2}{\pi(\hbar V)^2}\underbrace{\int_0^\infty \frac{dz\, z}{e^z+1}}_{\text{A number}}$

Thus, $n_e \propto T^2$

The average electron energy

$$\mathcal{E} = \int_0^\infty \frac{d\varepsilon\, \varepsilon g_e(\varepsilon)}{e^{\frac{\varepsilon}{T}}+1} = \frac{T^3}{\pi(\hbar V)^2}\int_0^\infty \frac{dz\, z^2}{e^z+1}$$ Another number

Thus, the specific heat is proportional to T^2

Part II

$$g_e(\varepsilon) = 2\int \frac{d^3p}{(2\pi\hbar)^3}\,\delta(\varepsilon - \varepsilon_e(p)) = \frac{(2m_e)^{3/2}}{2\pi^2\hbar^3}\sqrt{\varepsilon}$$

The density of the hole states $g_h = \frac{(2m_h)^{3/2}}{2\pi^2\hbar^3}\sqrt{\varepsilon}$

The position of the Fermi level is determined from the electroneutrality condition

$$\int_0^\infty d\varepsilon \frac{g_e(\varepsilon)}{e^{\frac{\varepsilon-\mu}{T}}+1} = \int_0^\infty d\varepsilon \frac{g_h(\varepsilon)}{e^{\frac{\varepsilon+\mu}{T}}+1} \rightarrow \int_0^\infty d\varepsilon \frac{\sqrt{\varepsilon}}{e^{\frac{\varepsilon-\mu}{T}}+1} = \left(\frac{m_h}{m_e}\right)^{3/2}\int_0^\infty d\varepsilon \frac{\sqrt{\varepsilon}}{e^{\frac{\mu+\varepsilon}{T}}+1} \quad (*)$$

Large number

Thus $\mu \gg T$, so that the left-hand-side of Eq. (*) is $\frac{2}{3}\mu^{3/2}$, while the right-hand-side is $\left(\frac{m_h}{m_e}T\right)^{3/2}\sqrt{\pi}\,e^{-\frac{\mu}{T}}$. Therefore, $\mu \approx \frac{3}{2}T\ln\left(\frac{m_h}{m_e}\right)$ and $n_e \propto T^{3/2}$