Common Exam Prep: Statistical Physics, July 25, 2007

(a) Assuming Maxwell-Boltzmann distribution for gases, find an expression for the average x-velocity, $\overline{v}_{+x}$, of THOSE PARTICLES MOVING IN THE $+x$ direction. Express your answer in terms of the molecular mass m and the temperature T.
(b) The density of air molecules at room temperature and sea-level pressure is about 2.7×10^{25} molecules/m^3. Use this and your solution to part (a) to calculate the flux (in units of molecules/m^2/s) of particles moving in any ONE direction past any point in your dorm room at the Starfleet Academy (in San Francisco, which is roughly at sea level).
(c) If a Delta-quadrant micro-meteorite punctured a hole 0.2 mm in diameter in the wall of the U.S.S. Voyager, at what rate (in units of molecules/s) would molecules leave if the air in the starship were regulated at sea-level pressure and room temperature, and the hull breach containment force-field is down.

(a) $Z_{tr} = \int e^{-\frac{mv^2}{2\kappa T}} d^3v =$

$$= \int_0^{\infty} e^{-\frac{mv_x^2}{2\kappa T}} dv_x \cdot \int_{-\infty}^{+\infty} e^{-\frac{mv_y^2}{2\kappa T}} dv_y \cdot \int_{-\infty}^{+\infty} e^{-\frac{mv_z^2}{2\kappa T}} dv_z$$

$$\bar{v}_x = \frac{\int_0^{\infty} v_x e^{-\frac{mv_x^2}{2\kappa T}} dv_x \cdot \int_{-\infty}^{+\infty} e^{-\frac{mv_y^2}{2\kappa T}} dv_y \cdot \int_{-\infty}^{+\infty} e^{-\frac{mv_z^2}{2\kappa T}} dv_z}{\int_0^{\infty} e^{-\frac{mv_x^2}{2\kappa T}} dv_x \int_{-\infty}^{+\infty} e^{-\frac{mv_y^2}{2\kappa T}} dv_y \int_{-\infty}^{+\infty} e^{-\frac{mv_z^2}{2\kappa T}} dv_z} =$$

$$= \frac{\int_0^{\infty} v_x \cdot e^{-\frac{mv_x^2}{2\kappa T}} dv_x}{\int_0^{\infty} e^{-\frac{mv_x^2}{2\kappa T}} dv_x} = \left(\frac{2\kappa T}{\pi m}\right)^{1/2}$$

b) There are 6 possibilities for the molecules

x ⇉
x ⇇
y ⇉
y ⇇
z ⇉
z ⇇

=> in one particular direction :

$$Flux = \frac{1}{6} \rho \cdot \bar{v}_x \simeq 1 \cdot 10^{27} \frac{molec}{m^2 \cdot s}$$

d) $$Rate = Flux \times \frac{\pi d^2}{4} \simeq 3.4 \cdot 10^{19} \frac{molec}{s}$$