Common Exam Prep Question
June 13, 2007

A. Consider an infinite metallic strip, $|y| < h$, in the xy-plane (Fig. 1). The strip carries a current, I. Neglecting the thickness of the strip, calculate the magnetic field, B, created by the current, as a function of y.

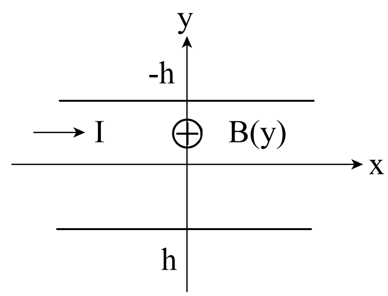


Fig. 1

B. A cylindrical beam of radius r_o, consists of particles with concentration, n, moving with velocity, v (Fig. 2). The Lorentz force causes the shrinkage of the beam. Estimate the length, L, at which the beam shrinks twice. (The charge of the particles is e and the mass is m.)

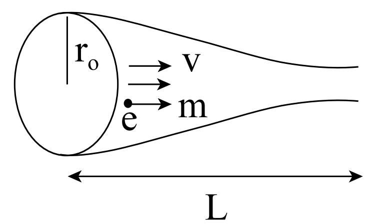


Fig. 2

Solution

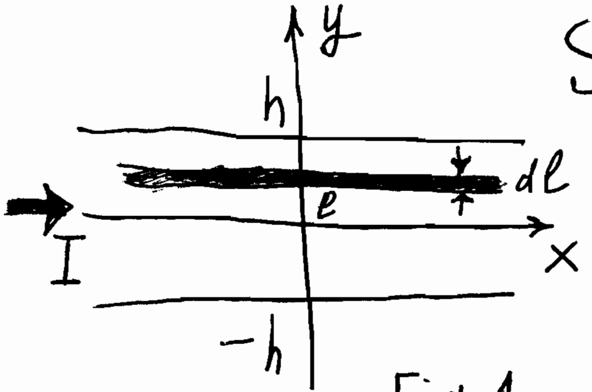


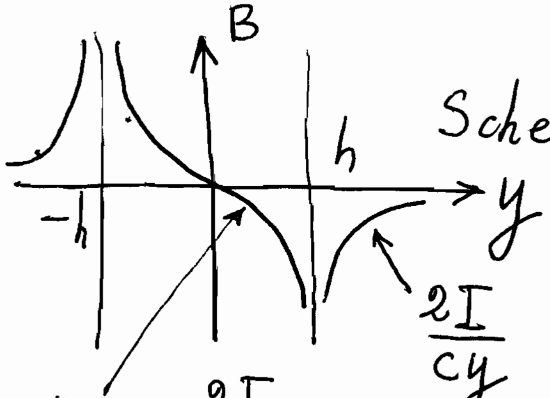
Fig. 1

Part I

Element, dl, carries current $dI = \frac{I}{2h}dl$ (Fig. 1). Magnetic field, created by this element at point, y, in the (x,y) plane is perpendicular to the (x,y) plane and is equal to $dB = \frac{2dI}{c(y-l)} = \frac{I\,dl}{ch(y-l)}$. Integrating over l, we obtain

$$B(y) = \frac{I}{ch}\int_{-h}^{h}\frac{dl}{y-l} = \frac{I}{ch}\ln\left|\frac{h-y}{h+y}\right|$$

Schematic behavior of $B(y)$ is shown in Fig. 2.



$B(y) = \frac{2I}{ch^2}y$

Fig. 2

Part II

$$B(r_0) = \frac{2I(r_0)}{cr_0} \qquad I(r_0) = J\pi r_0^2$$

Lorentz force $\qquad J = env$ ← current density

$$F = \frac{e}{c}B(r_0)v = \frac{e}{c}v\cdot\frac{2I}{cr_0} = \frac{2ev}{c^2r_0}(env)\pi r_0^2 = \frac{2\pi ne^2}{c^2}r_0v^2$$

Time during which the particle is deflected by $\frac{r_0}{2}$ (t)

$$\frac{Ft^2}{m} = r_0 \rightarrow t = \sqrt{\frac{mr_0}{F}} \rightarrow L = vt = \sqrt{\frac{mr_0}{F}v^2}.$$

Substituting $F = \frac{2\pi ne^2}{c^2}r_0v^2$, we get $L = \sqrt{\frac{mc^2}{2\pi ne^2}}$ → Note that r_0 and v do not enter into L.