A. Consider an infinite metallic strip, \(|y| < h\), in the xy-plane (Fig. 1). The strip carries a current, \(I\). Neglecting the thickness of the strip, calculate the magnetic field, \(B\), created by the current, as a function of \(y\).

\[
\begin{array}{c}
\text{\(y\)} \\
\hline
\text{-h} \\
\hline
\text{I} \\
\hline
\text{h} \\
\hline
\text{x}
\end{array}
\]

Fig. 1

B. A cylindrical beam of radius \(r_o\), consists of particles with concentration, \(n\), moving with velocity, \(v\) (Fig. 2). The Lorentz force causes the shrinkage of the beam. Estimate the length, \(L\), at which the beam shrinks twice. (The charge of the particles is \(e\) and the mass is \(m\).)

\[
\begin{array}{c}
r_o \\
\hline
v \\
\hline
e \\
\hline
m \\
\hline
L
\end{array}
\]

Fig. 2
Solution

Part I

Element, dl, carries current
\[ dI = \frac{I}{2h} \, dl \, (\text{Fig. 1}). \]

Magnetic field, created by this element at point, y, in the (x,y) plane is perpendicular to the (x,y) plane and is equal to
\[ dB = \frac{2dI}{c(y-l)} = \frac{I \, dl}{ch(y-l)}. \]

Integrating over \( l \), we obtain
\[ B(y) = \frac{I}{ch} \ln \left( \frac{h-y}{h+y} \right). \]

Schematic behavior of \( B(y) \) is shown in Fig. 2.

Part II

\[ B(r_o) = \frac{2I(r_o)}{cr_o} \quad \Gamma(r_o) = \frac{J \pi r_o^2}{c^2} \]

Lorentz force
\[ F = \frac{e}{c} B(r_o) V = \frac{e}{c} V \cdot \frac{2I}{cr_o} = \frac{2eV}{c^2r_o} (\text{env}) \pi r_o^2 = \frac{2\pi ne^2}{c^2} r_o V^2 \]

Time during which the particle is deflected by \( \frac{r_o}{2} \)
\[ \frac{Ft^2}{m} = \frac{r_o}{2} \rightarrow t = \sqrt{\frac{m r_o}{F}} \rightarrow L = V t = \sqrt{\frac{m r_o V^2}{F}}. \]

Substituting \( F = \frac{2\pi ne^2}{c^2} r_o V^2 \), we get \( L = \sqrt{\frac{m c^2}{2\pi ne^2}} \rightarrow \text{Note that } r_o \text{ and } V \text{ do not enter into } L. \]