A. Two particles with masses $m_1$ and $m_2$ attract each other with a gravitational force. At time $t = 0$ both particles are at rest in positions $x = x_1$ and $x = x_2$, respectively (Fig. 1). Find the time, $T$, after which the particle will stick together. Find the coordinate of the point at which they will meet.

B. Replace the particles with two sticks of length, $L$ (Fig. 2) Sketch the gravitational force, $F$, between the sticks as a function of separation, $x_2 - x_1$. Find $F$ in the limits $L << x_2 - x_1$, and $L >> x_2 - x_1$. For the latter limit estimate the time, $T$. 
Solution Part I

The center of mass remains at rest as the particles move. They will meet at 
\[ x = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \]

Equation of motion for the relative coordinate \( r \), reads \( \ddot{r} = \frac{F}{m} \), where 
\[ m = \frac{m_1 m_2}{m_1 + m_2} \]
and 
\[ F = G \frac{m_1 m_2}{r^2} \]

Energy conservation 
\[ \frac{\dot{r}^2}{2} - \frac{C}{r} = -\frac{C}{r_0} \]

where 
\[ r_0 = X_2 - X_1 \]

Estimate for \( T \): 
\[ \frac{\dot{r}^2}{2} \sim \frac{C}{r_0} \]

\[ T_0 = \left( \frac{m r_0^3}{C} \right)^{1/2} = \frac{r_0^{3/2}}{\left[ G(m_1 + m_2) \right]^{1/2}} \]

\[ r_2 = \frac{r_0 Z}{\sqrt{1 - Z}} \]

\[ \tau = \frac{\pi}{2\sqrt{2}} \]

\[ T = \frac{\pi}{2\sqrt{2}} T_0 = \frac{\pi}{2\sqrt{2}} \frac{r_0^{3/2}}{\left[ G(m_1 + m_2) \right]^{1/2}} \]

Part II

\[ F = \frac{G m_1 m_2}{r^2} \]

\[ \frac{\dot{r}^2}{2} \sim \frac{C}{L} \]

\[ T_0 = \left( \frac{m r_0^2 L}{C} \right)^{1/2} \]