Shown in Fig. 1 is the cross section of a conductor infinite in the z-direction and carrying a current, I (current flows in the dashed area). Assuming that $d \ll b \ll a$, find the magnitude and direction of the magnetic field created by the current at points $M_1$, $M_2$, and $M_3$, and at the coordinate origin.

Point coordinates:

- $M_1$: $y = 0$, $x = a + x_1$, where $b \ll x_1 \ll a$
- $M_2$: $y = 0$, $x = a + x_2$, where $x_2 >> a$
- $M_3$: $x = 0$, $y = b + y_3$, where $y_3 \ll b$

Fig. 1
Solution

General relation: \( \int \vec{B} \cdot d\vec{l} = \frac{4\pi}{c} I_l \)

where \( I_l \) is the current \( l \) flowing through the contour \( \vec{l} \). The field at the origin is zero from symmetry. For point \( M_3 \) we choose a contour \( l \):

\[ I_l = \frac{I l}{4b + 4a} \rightarrow 2B l = \frac{4\pi}{c} \frac{I l}{4b + 4a} \]

\[ B = \frac{\pi I}{M_3 c (2b + 2a)} \]

does not depend on \( y \) and is parallel to \( x \)-axis. For point \( M_2 \) we choose a contour, which is a circle with radius, \( a + x_2 \):

Then \( B = \frac{2\pi (a + x_2)}{M_2} \)

\[ B = \frac{2I}{c (a + x_2)} \]

parallel to the \( y \)-axis

For point \( M_1 \) we notice that elementary cross section \( dx \) creates magnetic field \( dB \):

\[ dB = \frac{2I}{c (b + a)} \left( \frac{2I}{4b + 4a} \right) dx \]

Then

\[ B = \frac{I}{M_2 c (b + a)} \int_{x_1}^{x_1 + 2a} \frac{dx}{x} = \frac{I}{c (b + a)} \ln \left( \frac{x_1 + 2a}{x_1} \right) \]