Show your reasoning clearly. If you are using a physical law or principle, always state what it is!!!!
Use the conversion constants and data given on the front page.
Not all given data may be necessary to solve the problems.

A. [9 points; explanation required] A newly discovered comet-like object is observed to make a pass around the sun. What would you measure to tell if the object will return many years later, or if it will never return?

The object will never return if the mechanical energy of the object-sun system is greater than or equal to zero

\[ \varepsilon \geq 0 \Rightarrow \frac{1}{2} m v^2 - \frac{G M m}{r} \geq 0 \Rightarrow \frac{1}{2} m v^2 \geq \frac{G M m}{r} \]

Since \( m \) cancels, you would need to measure the velocity of the object and its distance from the sun's center.

B. [8 points; explanation required] A round metal hoop is suspended on the edge by a hook. The hoop can oscillate side to side in the plane of the hoop, or it can oscillate back and forth in a direction perpendicular to the plane of the hoop. For which mode will the frequency of oscillation be larger?

1. Oscillations in the plane of the hoop.
2. Oscillations perpendicular to the plane of the hoop.
3. The frequency of oscillation will be the same in either mode.

\[ I_1 = m r^2 + m r^2 = 2 m r^2 \]

\[ I_2 = \frac{1}{2} m r^2 + m r^2 \quad \text{parallel axis theorem} \]

\[ I_1 > I_2 \quad \text{so} \quad \omega_1 < \omega_2 \]

C. [8 points; explanation required] Two radio waves moving along the same wave guide are defined by \( y_1 = 2 \sin(kx - \omega t + 0) \) and \( y_2 = 2 \sin(kx - \omega t + 2\pi) \). What is the amplitude of the resultant wave?

The new wave is a superposition of the two waves.

\[ Y = y_1 + y_2 = 2 \sin(kx - \omega t) + 2 \sin(kx - \omega t + 2\pi) \]

Since \( \sin(k + 2\pi) = \sin k \)

\[ Y = 2 \sin(kx - \omega t) + 2 \sin(kx - \omega t) = 4 \sin(kx - \omega t) \]

Amplitude equals 4
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A ring of matter is a familiar structure in planetary and stellar astronomy. Examples include Saturn's rings and a ring nebula. Consider a uniform ring of mass $M = 2.36 \times 10^{20}$ kg and radius $r = 1.00 \times 10^8$ m. An object of mass $m = 1,000$ kg is placed at a point $A$ on the axis of the ring, $d = 2.00 \times 10^8$ m from the center of the ring (see figure). When the object is released, the attraction of the ring makes the object move along the axis toward the center of the ring (point $B$).

(a) \[9 \text{ pts.}\\] Calculate the gravitational potential energy of the object-ring system when the object is at $A$.

(b) \[8 \text{ pts.}\\] Calculate the gravitational potential energy of the system when the object is at $B$.

(c) \[8 \text{ pts.}\\] Calculate the speed of the object as it passes through $B$ (neglect any motion of the ring).

\[a) \quad U = - \frac{G M m}{r} = - \frac{6 \times 10^{20} \times 1000}{1.00 \times 10^8} = -70397 \text{ J} \]

\[b) \quad at \quad b \quad r = r \quad since \quad d = 0 \quad U = - \frac{6 \times 10^{20} \times 1000}{2.00 \times 10^8} = -157412 \text{ J} \]

\[c) \quad Using \ conservation \ of \ energy \ for \ the \ ring-object \ system \]
\[k_i + U_i = k_f + U_f \quad k_i = 0 \]
\[k_f = \frac{1}{2} m v_f^2 = U_i - U_f = U_B - U_A \]
\[v_f = \sqrt{k_f \cdot m} = 13.2 \text{ m/s} \]
Show your reasoning clearly. If you are using a physical law or principle, always state what it is!!!!!
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A block of mass \( m = 0.3 \text{ kg} \) is thrown along a frictionless surface onto a vertical platform connected to a wall by a spring with spring constant \( k = 4.8 \text{ N/m} \). The mass of the platform and spring are negligible compared with \( m \). The speed of the block when it hits the platform is \( v = 2 \text{ m/s} \), and the block sticks to the platform as soon as it touches it.

(a) \( [8 \text{ pts.}] \) Find the maximum compression of the spring.
(b) \( [8 \text{ pts.}] \) How long does it take to reach the maximum compression point for the first time after the block hits the platform?
(c) \( [9 \text{ pts.}] \) Write the expression for the position of the block as a function of time during the oscillatory motion of the block and platform. (Use the form \( x(t) = A \cos(\omega t + \phi) \) and insert the explicit values of \( A, \omega, \text{ and } \phi \).

A) Conservation of Energy

\[
\frac{1}{2} m V_i^2 = \frac{1}{2} k x_f^2
\]

\[
\frac{1}{2} (0.3)(2)^2 = \frac{1}{2} (4.8) A^2
\]

\[
\Rightarrow A = 0.5 \text{ m}
\]

B) time to reach max compression @ \( \frac{T}{4} \) where \( T \) is the period: \( T = \frac{2\pi}{\omega} \)

\[
\omega = \sqrt{\frac{k}{m}} = 4 \frac{\text{rad}}{\text{s}}
\]

\[
\Rightarrow T = \frac{2\pi}{4} \text{ s} \quad \text{and} \quad t = \frac{T}{4} = \frac{\pi}{8} \text{ s} \approx 3.9 \text{ sec}
\]
c) \( A = 0.5 \) from part A
\( w = 4 \) from part B

Find \( \phi \):

\[
x(t) = A \cos(\omega t + \phi) \quad (1)
\]
\[
v(t) = -\omega A \sin(\omega t + \phi) \quad (2)
\]

@ \( t=0 \) \( x(0) = 0 \) \( m \)
and \( v(0) = -2 \) \( m/s \)

\[
x(0) = 0.5 \cos(4\cdot0 + \phi) \quad (3)
\]
\[
v(0) = -(4)(0.5) \sin(0 + \phi) \quad (4)
\]

(1) \( \phi = \cos^{-1}(0) = \frac{\pi}{2} \) \( or \) \( \frac{3\pi}{2} \)

(2) \( \phi = \arcsin(-1) = \frac{\pi}{2} \)

\( \Rightarrow \) \( \phi = \frac{\pi}{2} \)

So, we have

\[
x(t) = 0.5 \cos(4t + \frac{\pi}{2})
\]
Show your reasoning clearly. If you are using a physical law or principle, always state what it is!!!!!

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The wave function \( y(x, t) \) for a certain standing wave in a 2.51-m-long string fixed at both ends is given by 
\( y(x, t) = (0.05 \text{ m}) \sin(2.5 \times x) \cos(500 \times t) \), where \( x \) is in meters and \( t \) is in seconds.

(a) [6 pts.] What is the distance between successive nodes on the string?

(b) [6 pts.] What is the period \( T \) of the vibration?

(c) [8 pts.] Sketch the position of the string at the times \( t = 0 \), \( t = T/4 \), \( t = T/2 \), and \( t = 3T/4 \).

(d) [5 pts.] Find the fundamental frequency of the string.

(a) 
\[
\lambda_{NN} = \frac{1}{2} \quad \therefore \quad k = \frac{2\pi}{\lambda} \quad \Rightarrow \quad \lambda_{NN} = \frac{1}{2} \times \frac{2\pi}{k} = \frac{\pi}{2.5} = 1.26 \text{ m}
\]

(b) 
\[
T = \frac{2\pi}{\omega} = \frac{2\pi}{500} = \frac{\pi}{250} = 0.0126 \text{ s}
\]

(c) 
\[
y(x, t) = 2A \sin(kx) \cos(\omega t) \quad \text{where} \quad \omega = \frac{2\pi}{T}
\]

\[
y(x, t) = 2A \sin(kx) \cos\left(\frac{2\pi}{T} t\right)
\]

\[
at \quad t = 0: \quad y(x, 0) = 2A \sin(kx)
\]

\[
at \quad t = \frac{T}{4}: \quad y(x, \frac{T}{4}) = 0
\]

\[
at \quad t = \frac{T}{2}: \quad y(x, \frac{T}{2}) = -2A \sin(kx)
\]

\[
at \quad t = \frac{3T}{4}: \quad y(x, \frac{3T}{4}) = 0
\]

(d) 
\[
F_n = n \frac{V}{2L} \quad \Rightarrow \quad F_1 = \frac{1}{2L} \frac{V}{\omega} = \frac{1}{2} \times \frac{500}{2.5} = 40 \text{ Hz}
\]