Show your reasoning clearly. If you are using a physical law or principle, always state what it is!!!!
Use the conversion constants and data given on the front page.
Not all given data may be necessary to solve the problems.

A. [9 points] A satellite is circling around the moon (radius 1700 km) close to the surface at a speed \( v \). A projectile is launched from the moon vertically up at the same initial speed \( v \). How high will it rise from the surface of the moon?

\[
V_{\text{proj}} = \sqrt{\frac{GM}{r}} \quad \text{(USING CONSERVATION OF ENERGY)}
\]

\[
= \frac{v^2}{2g_{\text{moon}}} \quad ; \quad g_{\text{moon}} = \frac{GM}{r^2} \quad \Rightarrow \quad h = \frac{GM}{2} = \frac{r}{2} = 850 \text{km}
\]

B. [8 points] An aluminum casting is finished with a band sander. The sandpaper is stretched by two cylindrical rollers, one of them driven by a motor (see figure). The rollers have diameter of 16 cm. The coefficient of friction between aluminum and sandpaper is 1.2, and the force applied to push the casting against the sandpaper is 4 N. How large is the torque that needs to be applied to the driven cylinder?

\[
\tau = RF \quad ; \quad F = f = \mu N = 1.2 \times 4 = 4.8 \text{ N}
\]

\[
\Rightarrow \tau = \frac{0.16 \text{ m} \times 4.8 \text{ N}}{2} = 0.384 \text{ N.m}
\]

**NOTE:** To not confuse torque with energy, we write the units of torque as N.m. Normally a N.m is written as a Joule, as with energy.

C. [8 points] Two speakers separated by some distance emit sound waves of the same wavelength \( \lambda \), but the speakers are out of phase by 90°. Let \( r_1 \) be the distance from some point to speaker 1 and \( r_2 \) be the distance from that point to speaker 2. Find the smallest value of \( |r_2 - r_1| \) at which the sound at that point will be maximum. (Express your answer in terms of the wavelength \( \lambda \).)

\[
90° = \frac{\pi}{2} \quad ; \quad \Delta r = \left( \frac{\phi + 2N\pi}{2\pi} \right) \lambda
\]

\[
\Rightarrow \Delta r = \frac{\pi}{2} \quad \Rightarrow \lambda = \frac{\Delta r}{\pi}
\]
Show your reasoning clearly. If you are using a physical law or principle, always state what it is!!!!
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Some teenagers throw a rubber ball to a can on the top of a wall. The rubber ball, of mass \( m = 41 \text{ g} \), hits the can with a speed \( v = 1.80 \text{ m/s} \) at an angle of \( 30^\circ \) from the horizontal, and bounces back at an angle of \( 60^\circ \) from the same line (see figure). The can, of mass \( M = 71 \text{ g} \), is projected horizontally with speed \( v_0 \), and lands on the ground a distance \( D \) from the base of the wall. The height of the wall is \( H = 1.23 \text{ m} \).

\[(a) \quad [8 \text{ points}] \text{ Find the speed } v_0 \text{ of the can right after the collision.} \]
\[(b) \quad [7 \text{ points}] \text{ Find the amount of kinetic energy lost in the collision.} \]
Where did the energy go?
\[(c) \quad [5 \text{ points}] \text{ Find the time it takes the can to fall to the ground after the collision.} \]
\[(d) \quad [5 \text{ points}] \text{ Find the distance } D. \]

\[a) \quad \text{Momentum is conserved in } x \text{ and } y \text{ directions}\]
\[
x: \quad mv = Mv_0 \cos 30^\circ \implies v_0 = 1.2 \text{ m/s}\]
\[
y: \quad 0 = mv' - Mv_0 \sin 30^\circ \implies v' = 1.04 \text{ m/s}\]

\[b) \quad K_f - K_i = \left( \frac{1}{2} Mv_0^2 + \frac{1}{2} mv^2 \right) - \left( \frac{1}{2} mv' \right) = 0.007 \text{ J}\]

Energy \rightarrow \text{ sound, heat, etc.}\]

\[c) \quad \Delta y = v_{oy}t + \frac{1}{2} a t^2 \quad -H = 0 - \frac{a}{2} t^2 \quad t = \sqrt{\frac{2H}{a}} = 0.5 \text{ sec}\]

\[d) \quad \Delta x = v_{ox}t + \frac{1}{2} a t^2 \quad D = v_0 t + 0 \quad D = (1.2 \text{ m/s})(0.5 \text{ sec}) = 0.6 \text{ m}\]
A thin uniform rectangular sign hangs vertically above the door of a shop. The sign is hinged to a stationary horizontal rod along its top edge. The mass of the sign is $M = 2.40\, \text{kg}$ and its vertical dimension is $L = 50.0\, \text{cm}$. The sign is swinging without friction, becoming a tempting target for children armed with snowballs. The maximum angular displacement of the sign is $\theta_0 = 25.0^\circ$ on both sides of the vertical. At a moment when the sign is vertical and moving to the left, a snowball of mass $m = 400\, \text{g}$, traveling horizontally with a velocity $v = 1.60\, \text{m/s}$ to the right, strikes perpendicularly the lower edge of the sign and sticks there.

(a) [8 points] Calculate the angular speed of the sign immediately before the impact.

(b) [9 points] Calculate its angular speed immediately after the impact.

(c) [8 points] The spattered sign will swing up through what maximum angle?

A) CONSERVATION OF ENERGY ($E_i = E_f$)

$E_i = Mg(1 - \cos\theta) \frac{L}{2}$

$E_f = \frac{1}{2} I \omega^2$ ; $I = \frac{ML^2}{3}$

$Mg \frac{L}{2} (1 - \cos\theta) = \frac{1}{2} I \omega^2$

$\Rightarrow |\omega| = \sqrt{\frac{3g(1 - \cos\theta)}{L}} = 2.347\, \text{rad/s}$

B) CONSERVATION OF ANGULAR MOMENTUM ($L_i = L_f$)

$L_i = mvr - \frac{1}{3} MR^2\omega = mVL + \frac{1}{3} ML^2\omega = 0.149$

$L_f = I_{\text{tot}} \omega' = (mL^2 + \frac{1}{3} ML^2) \omega' = 0.3\, \omega'$

$mVL + \frac{1}{3} ML^2\omega = (mL^2 + \frac{1}{3} ML^2) \omega'$

$\omega' = \frac{mVL - \frac{1}{3} ML^2\omega}{mL^2 + \frac{1}{3} ML^2} = -0.498\, \text{rad/s} = 0.498\, \text{rad/s}$
c) CONSERVATION OF ENERGY \( (E_i = E_f) \)

\[
E_i = \frac{1}{2} I_{\text{tot}} \omega^2 = \frac{1}{2} \left( m l^2 + \frac{1}{3} M L^2 \right) \omega^2 = 0.037
\]

\[
E_f = (m+M)gh
\]

\[
\Rightarrow h = h_{\text{CM}} = \frac{\frac{1}{2} LM + Lm(1-\cos\theta)}{(M+m)} = 0.2857 \left( 1 - \cos\theta \right)
\]

\[
\frac{1}{2}(m l^2 + \frac{1}{3} M L^2) \omega^2 = (m+M)g \left( \frac{\frac{1}{2} LM + Lm}{M+m} \right) \left( 1 - \cos\theta \right)
\]

\[
\theta = \cos^{-1} \left[ 1 - \frac{\frac{1}{2} \omega^2 (m L^2 + \frac{1}{3} M L^2)}{g L \left( \frac{M}{2} + m \right)} \right] = 5.58^\circ
\]
Show your reasoning clearly. If you are using a physical law or principle, always state what it is!!!

Use the conversion constants and data given on the front page.

Not all given data may be necessary to solve the problems.

A level platform vibrates horizontally with simple harmonic motion with a frequency $f = 1.25$ Hz. A box rests on the platform, as shown in the figure, and the coefficient of static friction between the two is $\mu_s = 0.40$ (the coefficient of kinetic friction is $\mu_k = 0.35$).

(a) [6 points; explanation required] What is the magnitude of the force of friction when the platform passes through the equilibrium point?

(b) [7 points] What is the maximum amplitude of vibration if the box is not to slip?

(c) [6 points; explanation required] Assume the box does not slip. Does the force of friction on the box point toward or away from the equilibrium point?

(d) [6 points] If the box had started to slip, what would its acceleration had been while slipping?

**Part a.** At equilibrium, $\Sigma F = ma = 0$ because $\alpha = 0$. The only force that causes the box to experience an acceleration is the Frictional Force. So if $\alpha = 0$ then $F_f = 0$

**Part b.** The General Solution to the Harmonic Oscillator is $x(t) = A \cos(\omega t + \phi)$

$$\ddot{x}(t) = -\omega^2 A \cos(\omega t + \phi) \Rightarrow \dot{A}_{\text{max}} = \omega^2 A_{\text{max}}$$

Also, $\Sigma F = ma$ (on the Box) $\Rightarrow \dot{A}_{\text{max}} = \mu_s mg$

$$\Rightarrow A_{\text{max}} = \frac{\mu_s mg}{\omega^2}$$

So we have $A_{\text{max}} = \frac{(0.40)(9.81 m/s^2)}{(2\pi)^2 (1.25)^2} = 0.0636 m$.

**Part c.** If the box does not slip it is at rest with respect to the platform. This means that the only force that causes the Box to experience acceleration is the Static Friction Force. $\dot{\alpha}$ always points towards the equilibrium position. So Friction must also point in that same direction.

**Part d.** When slipping, on the Box, $\Sigma F = ma = F_{kx} = \mu_k mg$

$$a = \mu_k g = 0.35(9.81 m/s^2) = \frac{3.43 m}{s^2}$$