FIFTH MIDTERM -- REVIEW PROBLEMS

A solution set is available on the course web page in pdf format (no solutions for 27-29).

1. Assume that the planet Uranus has a radius of $2.50 \times 10^7$ m. A moon orbits the planet with a period of 30.0 days at a distance of $7.50 \times 10^7$ m above the surface of the planet.
   (a) Calculate the mass of Uranus from this data.
   (b) Find $g$ on the surface of Uranus.
   (c) Calculate the average density of Uranus.

2. (a) Find the value of $g$ 3000 miles above the moon's surface (ignore the Earth).
   (b) Find the value of $g$ 500 miles from the center of the moon.

3. (b) The Earth is $9.3 \times 10^7$ miles from the Sun. Calculate the gravitational force between the Earth and the Sun.
   (d) The Earth-Sun distance is called 1.00 A.U. (astronomical unit). If Jupiter orbits the Sun in 11.9 years, what is the Jupiter-Sun distance in A.U. (Assume circular orbits.)
   (e) If the mass of the Earth were doubled and its radius were doubled, by what fraction would your weight be multiplied?

4. (a) The mass of Jupiter is $1.88 \times 10^{27}$ kg. Its satellite, Io, has an orbital period of 42 hours. Find the radius of Io's orbit.
   (b) Given $g$ on the surface of the moon (.167 m/s$^2$) and a radius of $1.74 \times 10^7$ km, find the average density of the moon.
   (c) Find the value of $g$ 2000 miles above the surface of the moon.
   (d) A satellite is in orbit above the Earth. If the mass of the Earth were suddenly doubled, find the ratio $V_{new}/V_{old}$, giving the change in velocity of the satellite necessary to keep the orbit radius the same.

5. A certain hypothetical planet has a radius of 500 km and a surface gravity of 3.0 m/s$^2$.
   (a) What is the gravitational acceleration of an object 100 km above the surface of the planet?
   (b) What is the mass of the planet?

6. In the drawing shown, $M_1$ and $M_2$ are stars and $M_3$ is a planet. If $M_1 = 6.0 \times 10^{25}$ kg, $M_2 = 3.0 \times 10^{27}$ kg and $M_3 = 4.0 \times 10^{22}$ g, find the gravitational force (magnitude and direction) on $M_3$. Take $a = 8.0 \times 10^7$ km.

7. Two equal masses, $m_1$ and $m_2$ are situated a distance 2a apart along the y-axis.
   (a) Calculate the gravitational field at point P, a distance R from the midpoint of the two masses and on the y-axis.
   (b) Using the binomial expansion find the first nonzero term in the gravitational field that differs from the field of a mass 2m, situated at the midpoint.
8. Jupiter has a moon Io, whose period about the planet is 42.1 hours.

(a) If the orbit of Io has a radius of $4.22 \times 10^8$ m, find the mass of Jupiter.

(b) Derive Kepler's Third Law starting with Newton's Law of Gravity and the basic mechanics of circular motion.

9. The Earth circles the Sun in an orbit whose radius (take the orbit as circular) is 92,000,000 miles, in a time of 365 days. The diameter of the Sun is 860,000 miles. Find the mass and the average density of the Sun. (Be sure to show how you got these numbers.)

10. An astrophysicist is making a model of a star. He takes the mass as $M_o$ and the radius as $R_o$. He proposes that the density vary as $\rho(R) = \rho_o (1 - \alpha R^2)$.

(a) Find an expression for $\alpha$ in terms of $M_o$, $R_o$, and $\rho_o$.

(b) If a planet orbits this star at a distance of 100,000,000 miles with a period of 88 days, find the mass (in kg) of the star.

11. The radius of the Earth is 4000 miles. Take a model of the Earth in which an inner sphere of radius 2000 miles has an average density of $9.0 \times 10^3$ kg/m$^3$. Find the average density of the outer part of the Earth such that $g$ on the surface remains 9.80 m/s$^2$.

12. Consider a planet where the density varies according to the relationship $\rho = \rho_o (1 - \alpha R)$, where $\rho_o$ is the density at the center, $\alpha$ is a constant, and $R$ is the distance from the center. The radius of the planet is $R_o$.

(a) Calculate the mass of the planet.

(b) Calculate the value of $\mathbf{\vec{g}}$ at a point below the surface of the planet, which is a distance $R_o/3$ from the center of the planet.

13. (b) Assume the planet Mercury orbits the Sun 90.0 days at a distance of 30,000,000 mi. Use this data to calculate the mass of the Sun in kg. It will not be the same as on the data sheet.

(d) Take the Earth-Moon distance as 240,000 mi. Calculate the gravitational force in Newtons between the Earth and the Moon.

14. A satellite of mass of 150 kg is in a circular orbit 8000 mi above the Earth's surface.

(a) Calculate the velocity of the satellite in this orbit.

(b) Calculate the total energy of the satellite in this orbit.

(c) Calculate the minimum amount by which the satellite's velocity must be increased in order that it escapes from the Earth. Ignore the Sun and all other planets.

15. (a) Calculate the speed (in m/s) of a satellite in a circular orbit 1000 mi above the surface of the Earth.

(b) Calculate the total energy of the satellite in (a) if its mass is 1500 kg.

(c) Calculate the gravitational force between two elephants (assumed to be spheres) if there is 6.00 m between their centers. Take the mass of each elephant as 4000 kg.

(d) Assume that a black hole has a total mass 3.00 times the mass of the Sun. Calculate the speed of a small object in a circular orbit 1000 miles from the center of the black hole.

16. The planet Pluto has a moon, Charon, whose orbital period is 6 days. Assume that the mass of Charon is small compared with the mass of Pluto (probably not true). Assume that the mass of Pluto is 0.10 $M_E$.

(a) Find the radius of Charon's orbit.

(b) If the radius of Pluto is 200 km, find the escape speed from its surface.

(c) Assume that Pluto is in a circular orbit of radius $5.90 \times 10^4$ km about the Sun. Calculate the orbital period of Pluto about the Sun.
17. (a) If a satellite circles the Earth in a circular orbit 500 km above the surface of the Earth in exactly 90 min, what is the mass of the Earth (not quite the same as on the data sheet)?

(b) Calculate the total energy of the Earth in its orbit around the Sun. Assume the radius of the orbit is $1.48 \times 10^8$ km.

18. Assume that a geologist wishes to make a model of the Earth where the density varies as $\rho(R) = \rho_o(1 - \alpha R^2)$. Assume that the radius of the Earth ($R_o$) and the mass of the Earth ($M_o$) are as given on the data sheet. Use $\rho_o = 7.50 \times 10^3$ kg/m$^3$.

(a) Find the value of $\alpha$.

(b) Calculate the value of $g$ at point $R = R_o/2$.

If you can't do (a), leave $\alpha$ in (b) as a symbol.

19. (a) If $g$ on the surface of Mars is 3.70 m/s$^2$ and the mass of Mars is $6.00 \times 10^{23}$ kg, calculate the radius of Mars.

(c) If the Moon has an average density of 3340 kg/m$^3$ and a radius of 1738 km, what is the mass of the Moon?

20. Captain Kirk and the Enterprise are in orbit about a planet whose radius they measure as 8250 km. When they are in a circular orbit a distance of 24,000 km above the surface of the planet, they measure the period of that orbit as 3.34 hrs. Calculate the density of the planet.

21. The density of a star is approximated by $\rho = \rho_o(1 - \alpha R^2)$, where $\alpha$ and $\rho_o$ are constants, and the density goes to zero at the surface, $R = R_o$. The radius of the star is $1.20 \times 10^9$ km, and its total mass is $8.00 \times 10^{30}$ kg.

(a) Calculate $\alpha$.

(b) Determine $\rho_o$.

(c) Find the value of $g$ at point $R_o/3$ (inside the star).

22. The Starship Enterprise is in a circular orbit about an unknown planet. The kinetic energy of the Enterprise is $1.00 \times 10^{19}$ J and its mass is $4.00 \times 10^7$ kg. The orbit is $14.0 \times 10^6$ m above the surface of the planet, whose radius is $7.00 \times 10^6$ m.

(a) Calculate the potential energy (numerical value) of the Enterprise in this orbit. Ignore influences other than this planet.

(b) Determine the total energy (numerical value) of the Enterprise.

(c) Find the average mass density of this planet.

23. A geophysicist decides to model the earth as having a mass density that diminishes linearly with the distance from the center using $\rho(r) = \rho_o - \alpha r$ where $\rho_o$ and $\alpha$ are constants and $r$ is the distance from the center of the earth.

(a) If he chooses $\rho_o = 1.00 \times 10^4$ kg/m$^3$ find the magnitude and units of $\alpha$ that gives the correct mass for the earth.

(b) Using the above form for the mass density, calculate the magnitude of the effective "$g$" at $R = 4.00 \times 10^3$ km from the center of the earth.

(c) If the radius of the earth were doubled and the mass of the earth increased by a factor of 10.0, what is the weight of a student who now weighs 1200 N?

(e) Calculate the speed (in m/s) of a satellite in a circular orbit 2000 km above the surface of the earth.
25. The planet Mercury orbits the sun in a period of 100.0 days at a distance of 40,000,000 miles (number are not quite the true value).

(a) Calculate the mass of the sun in kg using the above numbers.
(b) If the radius of the sun is 500,000 miles, find its mass density in kg/m$^3$ using the mass obtained in part (a).

26. For the following question, use the data from the data sheet and an earth-sun distance of 93,000,000 miles between their centers.

(a) Find the total energy (kinetic plus potential) of the earth in its orbit (assume a circular orbit).
(b) Calculate the total speed needed for the earth to leave its orbit and escape from the sun. That is, find the velocity at its present distance from the sun needed to escape.

27. (a) If the mass of the earth were doubled and its radius tripled, what would be the new weight of a person who now weighs 200 pounds?
(b) The period of Halley's comet about the sun is 76.0 years. Calculate the value for the semi-major axis of its elliptical orbit.
(c) Calculate "g" 1000 km above the surface of the moon.
(d) Calculate the period of a satellite in a circular orbit about the moon 1.0 km above the surface of the moon.

28. Planet Mercury orbits the sun in a circular orbit in 95.0 days at a distance of 6.00 $\times$ 10$^7$ km from the center of the sun (not the true values).

(a) Find the mass of the sun from the above data. (Do not use the data sheet.)
(b) If the average density of the sun is 1,400 kg/m$^3$, what is the sun's radius?
(c) Calculate the speed of Mercury in its orbit.

29. You make a model of the earth with the density varying as $\rho = \rho_o(1 - \alpha R^2)$, with $\rho_o = 8000$ kg/m$^3$. Given that the mass of the earth is 6.00 $\times$ 10$^{24}$ kg and the radius is 6.40 $\times$ 10$^6$ m, calculate:

(a) the value of $\alpha$,
(b) the value of "g" at a point, P, 1/3 of the distance from the center of the earth to the surface. If you failed to get a numerical answer (a), leave $\alpha$ as a symbol in your solution to (b).

30. (d) Find the ratio of the orbital speed for two satellites in circular orbits 7000 km above the center of the moon and above the center of the earth.
(e) A planet orbits the sun (circular orbit) in 75 years. In units of the distance between the sun and the earth [R(sun-earth) = 1 A.U.], how far away is the planet from the sun?
31. Two satellites are in circular orbits around the earth in the same plane. One satellite is 230 km above the surface of the earth and the other is $4.11 \times 10^3$ km above the surface of the earth. Their instantaneous velocities are indicated in Fig. 1. At $t = 0$ the two satellites are directly above one another as shown in Fig. 1.

(a) What is the shortest time after $t = 0$ required for the two satellites to be on opposite sides of the earth (where the centers of the two satellites and the earth are also all on the same line) as shown in Fig. 2?

(b) With respect to the orientation shown in Fig. 1, sketch where satellite 1 and satellite 2 are when they are on opposites sides of the earth.

(c) If both satellites have the same mass, determine which satellite has the largest total energy. Fig. 1

\[ \text{Fig. 1} \quad \text{Fig. 2} \]

32. (c) A 1.00 kg air cart similar to the one used in class is moving horizontally at a velocity of 0.256 m/s to the left. The cart hits a second 0.500 kg cart moving at a speed of 0.128 m/s in the same direction. If the two carts stick together, find the final velocity (speed and direction).

(d) Find the internal kinetic energy of the system in part (c) before and after the collision, $k_i$ and $k_f$, respectively. If you have not solved part 9c), use $v$ for the final velocity.

(e) A particle in space of mass $m_1 = 2.00$ kg is moving with velocity $v = 8$ m/s î. The particle breaks up into two particles as shown in the diagram, where $m_1 = 1$ kg and $m_2 = 1.00$ kg. There are no net external forces. Find $V_{m_2}$ (magnitude of direction).

33. A satellite orbits the moon $8.30 \times 10^4$ m above the surface. For half of this orbit the satellite is out of communication with the earth. How long is this time?

34. Calculate the center of mass of the object in the figure. Assume the object is a uniform sheet of thickness $t$ and density $\rho$. Give both $x$ and $y$ coordinates for the center of mass.

35. An organ pipe is open at both ends. By experiment, resonances are found at 655 Hz, 1048 Hz and 2227 Hz as well as others. Take the speed of sound as 330 m/s.

(a) What is the largest value of the fundamental frequency allowed by the data? (None of the above is the fundamental.) Show clearly that all of the frequencies above are related to the fundamental you calculate in the correct manner.

(b) If there is a pressure node at each open end of the pipe, how many pressure nodes are there between the ends for the frequency 1703 Hz?

(c) How long is the pipe?
36. A violin type string is clamped between supports 27.0 cm apart. The string between the supports has a mass of 0.0150 kg, and the fundamental frequency is tuned to be 440 Hz.

(a) What is the tension needed in the string?
(b) What is the wavelength and frequency of the mode with 5 nodes between the clamps?
(c) Using the same string and the same tension, the positions of the clamps are changed. The frequency of the new first overtone (one node between the clamps) is found to be 1012 Hz. How far apart are the clamps?

37. An organ pipe is open at one end and closed at the other. By experiment, resonances are found at 635 Hz, 889 Hz, 1143 Hz and 1651 Hz, as well as others. Take the speed of sound as 330 m/s.

(a) What is the largest value of the fundamental frequency allowed by the data? (None of the above is the fundamental. The fundamental is lower than 635 Hz.)
(b) What is the length of the pipe.

38. A string is clamped between two supports 1.75 m apart. The total mass of the string free to oscillate between the supports is 0.0625 kg. The string is driven by a fixed frequency oscillator at 120 Hz.

(a) Calculate the values of the tension needed to produce the first four harmonics (the fundamental and the first three overtones) at the driving frequency.
(b) A new string is clamped in the same apparatus. It is found by experiment that the tension to produce the fundamental is 82.0% of that in (a). Find the total mass of the new string.

39. Given a wire 2.500 m long, whose mass is 0.150 kg. This wire is stretched between supports 2.500 m apart. Tension is supplied until the third harmonic standing wave occurs (2 nodes between the supports).

(a) Find the tension necessary to produce this result if the frequency of excitation is 125 Hz.
(b) Find the frequency of the fourth harmonic (3 nodes between supports) if all the conditions remain the same.

40. While singing in the shower, we notice that the system is resonant at certain frequencies. Consider only the end walls that are 8.00 ft apart (i.e., ignore effects due to side walls, ceiling and floor.)

(a) Calculate the first four frequencies at which resonant standing waves would occur between these walls. Assume the air is at 20.0°. \( V_{\text{sound}} = 1128.6 \text{ ft/s} \)
(b) If turning on the shower and increasing the humidity lowers the density of the air by 2.5%, leaving all other quantities unchanged, find the new fundamental resonance frequency. (A numerical value for the density of air is not needed.)

41. A tuning fork placed over an open vertical tube partly filled with water causes strong resonances when the water surface is 8 cm and 28 cm from the top of the tube and for no other positions. The speed of sound in the air in the room is 330 m/s. What is the frequency of the tuning fork?

42. An organ pipe is open at one end and closed at the other. It is adjusted so that the first overtone (the first resonant frequency higher than the fundamental) occurs when the tube is 2.25 m long. Take the velocity of sound to be 330 m/s. **Report numerical answers to three significant figures.**

(a) Find the wavelength of the first overtone.
(b) Find the frequency of the first overtone.
(c) Find the frequency of the fundamental.
(d) If the velocity of sound is increased by 1.00%, find the new frequency of the first overtone.

43. A string of 1.34 m in length and clamped at both ends, is excited at 250 Hz. The mass density of the string is 0.015 kg/m. Three nodes appear between the supports.

(a) Calculate the fundamental frequency.
(b) Calculate the tension in the string.
(c) Now the tension is increased until there are only two nodes between the supports for the same frequency of excitation. Find the new fundamental frequency and the new tension.
44. A violin string 0.700 m long is clamped at both ends. The mass of the string is 12.0 grams.

(a) Find the tension necessary so that the fundamental mode will be at a frequency of 440 Hz.
(b) Set the origin of your coordinate system (x = 0) at the midpoint of the string as shown. Write, as completely as possible, the function describing the waves on this string for the fundamental and the first two overtones. (Hint: Draw pictures of the waves for the three situations, and use these pictures to determine the appropriate functions.)

45. A violin string is given a tension of 200 N. The string has a mass density of 0.004 kg/m.

(a) Find the velocity of waves in this string.
(b) Find the wavelength of a 440 Hz wave in this string.
(c) If the string is attached between the supports 0.5 m apart, find a general formula for the allowed wavelengths in the system.
(d) For the three longest wavelengths you obtain in (c), find the frequency in Hz.

46. A violin string is tuned so that the fourth harmonic (three nodes between the supports) has a frequency of 1900 Hz. The tension in the string is 900 N. The supports are 0.30 m apart.

(a) Calculate the mass density of the string.
(b) If the tension is increased by 1.00%, calculate the frequency of the fundamental.
(c) Write a complete expression for the displacement y, as function of time and position, for the fourth harmonic described above. Choose x = 0 at the mid-point of the string.

47. A violin string is clamped between supports 0.45 m apart.

(a) If the string is tuned to a fundamental of 440 Hz with a tension of $2.75 \times 10^3$ N, calculate the mass density of the string.
(b) If the tension is increased to $2.85 \times 10^3$ N, calculate the new fundamental frequency.
(c) If it is desired to have the mode with two nodes between the supports be at a frequency of 1285 Hz, what tension is needed?

48. A tube is 1.45 m long and open at both ends. Resonances are found at 234 Hz, 585 Hz, 936 Hz, and 1404 Hz among others.

(a) Find the largest value of the fundamental allowed by these data.
(b) How many nodes for pressure are there for 1404 Hz--not counting the ends.
(c) Calculate the velocity of sound for this system.

49. A traveling wave is described by the function

$$y = (1.78 \text{ mm}) \sin \left( 27.0x + 5720t + \frac{\pi}{6} \right)$$

Except where shown, all distance are in meters. Other quantities are in the usual and appropriate units.

(a) Calculate the magnitude of the velocity of the wave
(b) Specify in words, the direction of the wave
(c) Calculate the wavelength
(d) Calculate the frequency in Hertz
(e) At $t = 0$, calculate the first positive value of x for which $y = 0$. 
50. An open tube is arranged with one end in a beaker of water. When a tuning fork (1024 Hz) is held nearby, resonances are observed for \(d = 3.00\) in, \(9.45\) in, \(15.90\) in and \(22.35\) in. Take the speed of sound as \(1100\) ft/s.

(a) For \(d = 15.90\) in, calculate the frequency of the fundamental. Include the "end correction" calculated from the information given.

(b) If helium is added to the tube so that its density is reduced from \(1.29\) g/l to \(1.17\) g/l, calculate the new value of the fundamental in (a). Consider only the change in density of the gas in the tube.

51. A wave on a string is described by the solution: \(y = (6.25 \times 10^{-3}) \sin (575x + 425t + 0.87)\). Distances are in meters, time in seconds, and other quantities have the usual and appropriate SI units.

(a) Calculate the velocity of the wave (including direction).
(b) Calculate the maximum value of the transverse velocity.
(c) At \(t = 0\), calculate the smallest positive value of \(x\) for which the displacement is \(+3.00 \times 10^{-3}\) m.
(d) Calculate the maximum value of the displacement.
(e) Calculate the frequency, \(f\) (the number of peaks per second passing a given point).

52. A wave on a string is described by the solution: \(y = (3.25 \times 10^{-3}) \cos (42.7x + 57.5t + 0.25)\). All distances are in meters, time in seconds, and all other quantities the usual and appropriate SI units.

(a) Calculate the velocity of the wave, including direction.
(b) Calculate the frequency, \(f\), in Hertz.
(c) Calculate the maximum value of the transverse velocity.
(d) Calculate the wavelength.
(e) Calculate the maximum value of the displacement.

53. A wave on a string is described by the function \(y = (4.75 \times 10^{-3}) \sin (7.50x + 32.0t - 0.35)\). Distances are in meters, time in seconds, and other quantities have the usual and appropriate SI units.

(a) Calculate the velocity of the wave (including direction).
(b) Calculate the maximum value of the transverse velocity.
(c) Calculate the maximum value of the displacement.
(d) Calculate the frequency \(f\) (the number of peaks per second past a given point).
(e) At \(x = 0\), calculate the smallest positive value of \(t\) (\(t > 0\)) for which the displacement has its maximum negative value at \(x = 0\).

54. A wave on a string can be described by the solution

\[y = (1.30 \times 10^{-3}) \sin (65.0x + 2400t)\]

All quantities have the usual and appropriate SI units.

(a) Calculate the velocity of the wave (including direction).
(b) Calculate the frequency (in Hertz).
(c) Calculate the wavelength.
(d) If the tension in the string is \(45\) N, calculate the mass density of the string.
(e) Calculate the first \textit{positive} value of \(t\) (\(t > 0\)) for which the displacement has its maximum \textit{negative} value at \(x = 0\).
55. The function below describes a wave traveling on a stretched string. (x is in meters and t in seconds.)

\[ y = (1.50 \text{ mm}) \sin(9.90x + 125t - \frac{\pi}{3}) \]

(a) Calculate the wavelength of the wave.
(b) Calculate the velocity of the wave, including its direction.
(c) Calculate the transverse velocity, giving its correct units and direction, for \( x = +2.00 \text{ m} \) and \( t = +3.00 \text{ s} \).
(d) Calculate the period \( T \) for the wave.
(e) If the string has a linear mass density of 0.020 kg/m, calculate the tension in the string.

56. A violin string with 30.0 cm between supports is tuned to a fundamental frequency of 440 Hz. The seventh harmonic is generated (six nodes between the supports not including those of the supports, i.e., 7 antinodes).

(a) Calculate the speed of the waves on the string.
(b) Find the frequency \( \text{AND} \) wavelength of the seventh harmonic.
(c) Determine the tension needed if the 30.0 cm of string has a mass of 17.0 gms.

57. Resonances are observed for an organ pipe open at one end and closed at the other for the following frequencies (none of these is the fundamental): 291 Hz; 679 Hz; 1067 Hz; 1649 Hz

(a) Calculate the largest value of the fundamental frequency (in Hz) permitted by this data.
(b) If the speed of sound is taken to be 330 m/s, calculate the effective length of this organ pipe.
(c) If helium is mixed with air to reduce its density by 10.0\%, determine the new value of the 291 Hz resonance considering \textit{only} the density change.

58. 1. A wave is described by \( y = (0.40 \text{ m}) \sin[(3.0 \text{ m}^{-1}) x + (2.0 \text{ s}^{-1}) t] \). Determine

(a) the wavelength \( \lambda \),
(b) the frequency \( f \),
(c) the amplitude \( A \),
(d) the period \( T \),
(e) the angular frequency \( \omega \), and
(f) the angular wave number \( k \).

2. If you take a grandfather clock to Mars, will it run faster or slower than on Earth? The diameter of Mars is 0.53 times the diameter of the Earth, and its mass is 0.11 times the mass of the Earth.

59. A standing wave on a string with fixed ends has four nodes (\textit{not} counting the ends). The frequency of this mode is 120 Hz.

(a) Sketch the shape of the wave.
(b) Find the fundamental frequency of the string.
(c) If the tension in the string is reduced by a factor of 9, what is the new fundamental frequency?

60. A uniform rod with a length \( L = 60 \text{ cm} \) and a mass \( M = 3.0 \text{ kg} \) is pivoted about one end and is hanging vertically down at rest. The free end of the rod is given an initial speed \( v = 0.30 \text{ m/s} \). Assume it undergoes simple harmonic motion in a vertical plane, and determine its

(a) period,
(b) total energy (let the potential energy be zero at the lowest point), and
(c) maximum angular displacement in degrees. \textit{(Note: the moment of inertia of a uniform rod about an axis through one end is \( I = \frac{1}{3} ML^2 \).}
61. Two stars of equal mass $m = 2.00M_{\odot}$ are revolving in a circular orbit about their center of mass (see figure). Assume the distance between the stars is $d = 10.0$ AU. (Here $M_{\odot} = 1.99 \times 10^{30}$ kg is the mass of the Sun and $1$ AU $= 1.50 \times 10^{13}$ m is one astronomical unit). Find

(a) the magnitude of the force of gravity exerted by one star on the other,
(b) the orbital speed in km/s, and
(c) the orbital period in days.

62. A. A block attached to a spring has simple harmonic motion with an amplitude of 4.0 cm. When the block is 2.0 cm from the equilibrium position, what fraction of its total energy is kinetic energy? Set the potential energy to zero at equilibrium. Give reasons for your answer.

(a) One-quarter          (b) One-third          (c) One-half           (d) Two-thirds       (e) Three-quarters

B. Some people think that shuttle astronauts are “weightless” because they are “beyond the pull of Earth’s gravity.” In fact, this is completely untrue.

(a) Assume the shuttle moves on a circular orbit 400 km above the ground. What is the magnitude of the acceleration of gravity for shuttle astronauts? The radius of the Earth is 6370 km. Use 3 significant figures.

(b) Given the answer in Part (a), why are shuttle astronauts “weightless”?

C. Two rectangular wave pulses are traveling at 10 cm/s in opposite directions along a string. At $t = 0$, the two pulses are as shown in the figure. Using the superposition principle, draw the wave functions for $t = 1$, $2$, and $3$ s.

63. Five objects of mass $M$ are equally spaced on the arc of a semicircle of radius $R$ as in the figure. An object of mass $m$ is located at the center of curvature of the arc.

(a) If $M$ is 3.0 kg, $m$ is 2.0 kg, and $R$ is 10. cm, what is the gravitational force on $m$ due to the five objects? Give both magnitude and direction.

(b) What is the gravitational potential energy of the object of mass $m$ in the field of the others? Set the zero of the potential energy at infinity; pay attention to its sign.
64. It is thought that the brain determines the direction of the source of a sound by sensing the phase difference between the sound waves striking the eardrums. A distant source emits a sound of frequency 680 Hz. When you are directly facing the source there is no phase difference. Assume that the distance between your ears is 20.0 cm and that sound travels at 330 m/s.

(a) Compute the wavelength of the sound wave.
(b) Compute the angular frequency and the angular wave number.
(c) Compute the phase difference between the sounds received by each ear when the source is exactly on your left (i.e., you have turned through 90° from facing directly toward the source).

65. An object of unknown mass \( m \) is hung on the end of a spring of unknown force constant \( k \). The object is held at rest with the spring unstretched. Then the object is released from rest and it falls 3.42 cm before turning around.

(a) Show that the object undergoes simple harmonic motion with period \( T = 2\pi \sqrt{\frac{m}{k}} \).
(b) Compute the period of the motion in seconds.
Data: Use these constants (where it states for example, 1 ft, the 1 is exact for significant figure purposes).

1 ft = 12 in (exact)
1 m = 3.28 ft
1 mile = 5280 ft (exact)
1 hour = 3600 sec = 60 min (exact)
1 day = 24 hr (exact)

\[ g_{\text{earth}} = 9.80 \, \text{m/s}^2 \]
\[ = 32.2 \, \text{ft/s}^2 \]

\[ g_{\text{moon}} = 1.67 \, \text{m/s}^2 \]
\[ = 5.48 \, \text{ft/s}^2 \]

1 year = 365.25 days
1 kg = 0.0685 slug
1 N = 0.225 pound
1 horsepower = 550 ft/pounds/s (exact)

\[ M_{\text{earth}} = 5.98 \times 10^{24} \, \text{kg} \]
\[ R_{\text{earth}} = 6.38 \times 10^3 \, \text{km} \]
\[ M_{\text{sun}} = 1.99 \times 10^{30} \, \text{kg} \]
\[ R_{\text{sun}} = 6.96 \times 10^{8} \, \text{m} \]
\[ M_{\text{moon}} = 7.35 \times 10^{22} \, \text{kg} \]
\[ R_{\text{moon}} = 1.74 \times 10^3 \, \text{km} \]
\[ G = 6.67 \times 10^{-11} \, \text{N} \cdot \text{m}^2/\text{kg}^2 \]
\[ m_{\text{electron}} = 9.11 \times 10^{-31} \, \text{kg} \]