Physics 172
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REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES.
Use the conversion constants and data given on the front page.

Assume that the planet Uranus has a radius of \(2.50 \times 10^7\) m. A moon orbits the planet with a period of 30.0 days at a distance of \(7.50 \times 10^7\) m above the surface of the planet. Calculate: (if you can’t do (a), do (b) and (c) algebraically)

(a) the mass of Uranus from this data.
(b) \(g\) on the surface of Uranus.
(c) the average density of Uranus.

\[
\begin{align*}
\text{a)} \quad \frac{v^2}{r} &= \frac{GM_u}{r^2} \quad \Rightarrow \quad v^2 &= \frac{GM_u}{r} \\
M_u &= \frac{4\pi^2 r^3}{T^2} \\
&= \frac{4\pi^2 (2.50 \times 10^7 + 7.50 \times 10^7)^3}{(2.59 \times 10^6)^2 (6.67 \times 10^{-11})} \\
&= 8.81 \times 10^{22}\ \text{kg}
\end{align*}
\]

\[
\begin{align*}
\text{b)} \quad g_u &= \frac{GM_u}{r_u^2} \\
&= \frac{(6.67 \times 10^{-11})(8.81 \times 10^{22})}{(2.50 \times 10^7)^2} \\
&= 9.40 \times 10^{-3}\ \text{m/s}\ ^2
\end{align*}
\]

\[
\begin{align*}
\text{c)} \quad \rho &= \frac{4}{3} \pi r_u^3 \\
&= \frac{M_u}{\frac{4}{3} \pi r_u^3} \\
&= \frac{8.81 \times 10^{22}\ \text{kg}}{\frac{4}{3} \pi (2.50 \times 10^7\ \text{m})^3} \\
&= 1.35 \times 10^6\ \text{kg/m}^3
\end{align*}
\]

-1 Sig Figs
-1 Units

04 4
PROBLEM 1B

1. (a) Find the value of g 3000 miles above the moon's surface (ignore the earth).

\[ g = \gamma \left( 10^{-2} \right)^2 \, \text{m/s}^2 \]

1. (b) Find the angular momentum of the earth rotating on its axis. \[ 2 \times 10^{33} \, \text{kg m}^2 \text{sec} \]

1. (c) Find the force between an electron and a proton 0.5 \times 10^{10} \text{ m} apart. \[ -\frac{e^2}{r} \, \text{N} \]

1. (d) Find the value of g 500 miles from the center of the moon. \[ 0.25 \, \text{m/s}^2 \]

1. (e) Find the electric field 20.0 \text{ m} away from a metal sphere which has a charge of 3.0 \times 10^9 \text{ C}.

\[ \frac{6 \times 10^{16}}{12} \, \text{V/C} \]

\[ \frac{G m}{r^2} \]

\[ \frac{2 \times (10^{-10}) \left[ (6 \times 10^{-10})(5 \times 10^{-10})(10.3) \right]^2 \cdot 2\pi}{24(60)(60)} \]

\[ = 7.3 \times 10^{33} \, \text{N m}^2 \text{ sec} \]

\[ F = \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{r^2} = \frac{9 \times 10^9 \left( 1.6 \times 10^{-19} \right)^2}{(0.5 \times 10^{-10})^2} \approx -9.2 \times 10^{-4} \text{ N} \]
(a) Calculate the electric force between a lithium nucleus \((z = 3)\) and an electron a distance \(1.00 \times 10^{-11}\) m away. 
\[-6.9 \times 10^{-6} \text{ N (attractive)}\]

(b) The earth is \(9.3 \times 10^7\) miles from the Sun. Calculate the gravitational force between Earth and Sun. 
\[3.5 \times 10^7 \text{ N}\]

(c) Calculate the electric field magnitude at a point midway between a charge of \(+7.0\) pC and one of \(-3.5\) pC. The distance is 0.75 m. 
\[0.67 \text{ N/C (to the right)}\]

(d) The Earth-Sun distance is called 1.00 A.U. (astronomical unit). If Jupiter orbits the Sun in 11.9 years, what is the Jupiter-Sun distance in A.U. (Assume circular orbits.) 
\[5.2 \text{ A.U.}\]

(e) If the mass of the Earth were doubled, and its radius were doubled, by what fraction would your weight be multiplied? 
\[\frac{1}{2}\]
d) Use \( T^2 = \frac{4\pi R^3}{GM} \) and form a ratio:

\[
\frac{T_s^2}{T_e^2} = \frac{4\pi R_s^3}{GM_s} \quad ; \quad \frac{T_e^2}{T_s^2} = \frac{4\pi R_e^3}{GM_e}
\]

\[
\frac{T_s^2}{T_e^2} = \frac{\frac{4\pi R_s^3}{GM_s}}{\frac{4\pi R_e^3}{GM_e}} = \frac{R_s^3}{R_e^3}
\]

\[
R_s = \left[ \left( \frac{R_s^3}{T_e^2} \right) \right]^{1/3} = \left[ \frac{(1 \text{ A.u.})^3 (11.9 \text{ yr})^2}{(1 \text{ yr})^2} \right]^{1/3}
\]

\[
R_s = 5.2 \text{ A.u.}
\]

d) \( F_3 = G \frac{M_e m}{R_e^2} \)

If \( M_e \to 2M_e \)

and \( R_e \to 2R_e \),

\[
F_3' = G \frac{(2M_e)m}{(2R_e)^2} = \frac{2}{4} G \frac{M_e m}{R_e^2} = \frac{1}{2} F_3
\]

The fraction is \( \frac{1}{2} \)
a) \( z = 3 \) means there are 3 protons in the nucleus, each with charge +e. Let nucleus = \( q^+ \), electron = \( q^- \).

\[
F_e = \frac{1}{4\pi\varepsilon_0} \frac{q^+ q^-}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{(3e)(-e)}{r^2} = -\frac{1}{4\pi\varepsilon_0} \frac{3e^2}{r^2}
\]

\[
= -(9.00 \times 10^9 \frac{N\cdot m^2}{C^2}) \frac{(3)(1.6 \times 10^{-19} C)^2}{(1.00 \times 10^{-11} m)^2}
\]

\[
F_e = -6.9 \times 10^{-6} N
\]

b) \( F_g = G \frac{M_s M_e}{r^2} \) We need \( M_s \):

\[
\frac{1}{r^2} = \frac{4\pi^2r^3}{GM_s} \Rightarrow M_s = \frac{4\pi^2r^3}{G} \quad r = 9.3 \times 10^7 \text{ mi}
\]

Converting to metric units \( \Rightarrow M_s = 1.90 \times 10^{30} \text{ kg} \)

Then \( F_g = (6.67 \times 10^{-11})(6.0 \times 10^{24} \text{ kg})(1.90 \times 10^{30} \text{ kg}) \)

\[
F_g = (6.67 \times 10^{-11}) \frac{(6.0 \times 10^{24} \text{ kg})(1.90 \times 10^{30} \text{ kg})}{[(9.3 \times 10^7 \text{ mi})\frac{5280 \text{ ft}}{\text{ mi}}\frac{0.30 \text{ m}}{\text{ ft}}]^2}
\]

\[
F_g = 3.5 \times 10^{22} N
\]

c) Both fields (one from \( \oplus \) and one from \( \ominus \)) are to the right:

\[
E = E_\oplus + E_\ominus = \frac{1}{4\pi\varepsilon_0} \left[ \frac{7.0 \times 10^{-12} C}{(d/2)^2} + 3.5 \times 10^{-22} C \right]
\]

\[
E = 0.67 \text{ N/C}
\]
[20 pts. - 5 pts. each]

(a) The mass of Jupiter is $1.88 \times 10^{27}$ kg. Its satellite, Io, has an orbital period of 42 hours. Find the radius of Io's orbit.

\[ \frac{GM}{r^2} = \frac{v^2}{r} = \frac{4\pi^2r}{T^2} \]

\[ r = \left( \frac{GM}{\frac{4\pi^2}{T^2}} \right)^{\frac{3}{2}} = \left( \frac{6.67 \times 10^{-11} \times 1.88 \times 10^{27} \times (42 \times 3600)^2}{4\pi^2} \right)^{\frac{3}{2}} = \frac{4\pi^2}{1.7 \times 10^9} \text{ m} \]

(b) Given \( g \) on the surface of the moon and the radius from the data sheet, find the average density of the moon.

\[ g = \frac{GM}{r^2}, \quad M = \frac{4\pi^2r^3}{3} g, \quad D = \frac{32}{3} \frac{g}{G} = \frac{3 \times 1.67}{4\pi^2 \times 6.67 \times 10^{-11} \times 1.74 \times 10^6} \approx 2.43 \times 10^3 \frac{kg}{m^3} \]

(c) Find the value of \( g \) 2000 miles above the surface of the moon. (Check data sheet.)

\[ \frac{g_2}{g} = \left( \frac{r + 2000}{r} \right)^2 \]

\[ g_2 = g \left( \frac{r + 2000}{r} \right)^2 \approx 3 \frac{1.67}{(2.3 \times 2.231 + 1.54 \times 10^3)^2} \approx 0.239 \frac{g}{g} \approx 216 \frac{m^2}{s^2} \]

(d) A satellite is in orbit above the earth. If the mass of the earth were suddenly doubled, find the ratio, \( \frac{V_{new}}{V_{old}} \), giving the change in velocity of the satellite necessary to keep the orbit radius the same.

\[ \frac{GM}{r^2} = \frac{v^2}{r}, \quad \frac{GM}{r} \quad \rho r = \text{const} \quad \text{so} \quad \frac{V_{new}}{V_{old}} = \sqrt{\frac{M_{new}}{M_{old}} = \sqrt{\frac{2m_{old}}{m_{old}}}} = \sqrt{2 : 1} \]
A certain hypothetical planet has a radius of 500 km and a surface gravity of 3.0 meters/sec². (a) What is the gravitational acceleration 100 km above the surface of the planet? (b) What is the mass of the planet?

(a) \( g_s = -\frac{GM}{r_s^2} \) \( g_r = -\frac{GM}{r^2} \), where \( r_s \) = radius of the planet

\( g_r = \left(\frac{r_s}{r}\right)^2 g_s = \left(\frac{500\, km}{600\, km}\right)^2 (3\, m/s^2) \)

\( g_r = 2.08\, m/s^2 \)

(b) \( M = \frac{g_s r_s^2}{G} = \frac{(3\, m/s^2)(500 \times 10^3\, m)^2}{(6.67 \times 10^{-11}\, \text{m}^3\text{kg}^{-1}\text{sec}^{-2})} \)

\( M = 1.12 \times 10^{22}\, \text{kg} \)
In the drawing shown, M\(_1\) and M\(_2\) are stars, and M\(_3\) is a planet. If M\(_1\) = 6.0 \times 10^{29} \text{ kg}, M\(_2\) = 3.0 \times 10^{27} \text{ kg}, and M\(_3\) = 4.0 \times 10^{22} \text{ kg}, find the gravitational force (magnitude and direction) on M\(_3\). Take a = 8.0 \times 10^7 \text{ km}.

\[ F_x = \frac{G M_2 M_3}{(a^2)^2} \cos 45^\circ \]
\[ = \frac{6.67 \times 10^{-11} \left(4 \times 10^{22}\right)\left(6.0 \times 10^{29}\right)}{(8 \times 10^7)^2} \cos 45^\circ \]
\[ = 8.76 \times 10^{19} \text{ N} \]

\[ F_y = \frac{G M_2 M_3}{(a^2)^2} \sin 45^\circ = \frac{6.67 \times 10^{-11} \left(4 \times 10^{22}\right)\left(6.0 \times 10^{29}\right)}{(8 \times 10^7)^2} \sin 45^\circ = 8.54 \times 10^{19} \text{ N} \]

Magnitude:
\[ F = \sqrt{F_x^2 + F_y^2} = \sqrt{(8.76 \times 10^{19})^2 + (8.54 \times 10^{19})^2} = 1.26 \times 10^{20} \text{ N} \]

3 SF because of the initial (1).

Direction:
\[ \theta = \tan^{-1} \left( \frac{F_y}{F_x} \right) = \tan^{-1} \left( \frac{8.54 \times 10^{19}}{8.76 \times 10^{19}} \right) = 44.6^\circ \text{ Below west} \]
\[ (25\text{F}) \]

or \[ 180 + 45^\circ = 225^\circ = 3.2 \times 10^2 \text{ degrees} \]

(Note that 220^\circ is 3 SF.)
To equal masses, \( m_1 \), are situated a distance \( 2a \) apart along the \( y \) axis.

(a) Calculate the gravitational field at point \( P \), a distance \( R \) from the mid-point of the two masses, and on the \( y \) axis.

(b) Using the binomial expansion find the first non-zero term in the gravitational field that differs from the field of a mass \( 2m \), situated at the mid-point.

**Solution:**

(a) \[
\mathbf{g} = -\frac{G m_1}{(R+a)^2} - \frac{G m_1}{(R-a)^2} = \frac{G m_1}{R^2} \left( \frac{1}{(R+a)^2} + \frac{1}{(R-a)^2} \right)
\]

(b) \[
\mathbf{g} = -\frac{G m_1}{R^2} \left( \frac{1}{(1+\frac{a}{R})^2} + \frac{1}{(1-\frac{a}{R})^2} \right)
\]

\[
= -\frac{G m_1}{R^2} \left( 1 - \frac{2a}{R} + \frac{3x^2 a^2}{2 R^4} + \ldots \right) + \left( 1 + \frac{2a}{R} + \frac{3x^2 a^2}{2 R^4} + \ldots \right)
\]

\[
= -\frac{G m_1}{R^2} \left[ 2 + \frac{6a^2}{R^2} + \ldots \right] = -\frac{G 2m}{R^2} - \frac{6G m a^2}{R^4}
\]

So that the first non-zero term which differs from the field of a mass \( 2m \), situated at the mid-point is

\[
\mathbf{g}^{(1)} = -\frac{6G m a^2}{R^4}
\]

Because the field of a mass \( 2m \), situated at the mid-point just is,

\[
\mathbf{g}^{(2)} = -\frac{G 2m}{R^2}
\]
Jupiter has a moon, Io, whose period about the planet is 42.1 hours.

(a) If the orbit of Io has a radius of $4.22 \times 10^8$ m, find the mass of Jupiter.

(b) Derive Kepler's Third Law starting with Newton's Law of Gravity, and the basic mechanics of circular motion.

\[ T^2 = \frac{4\pi^2 a^3}{G M_j} \]
\[ T = 42.1 \text{ hr} \left( \frac{3600 \text{ sec}}{1 \text{ hr}} \right) = 1.51 \times 10^5 \text{ sec} \]

\[ M_j = \frac{4\pi^2 a^3}{GT^2} = \frac{1.93 \times 10^{24} \text{ kg}}{1.51 \times 10^5} \]

\[ F = \frac{GM_i m_j}{R^2} = m_i \omega^2 R^2 \]
\[ \omega = \frac{2\pi}{T} \]

\[ \frac{GM_j}{R^2} = \left( \frac{2\pi}{T} \right)^2 \]
\[ T^2 = \frac{4\pi^2 R^3}{GM_j} \]

Showed as $4.22 \times 10^8$ m to get mass which agrees with Jupiter mass.
The Earth circles the Sun in an orbit whose radius (take the orbit as circular) is 92,000,000 miles, in a time of 365 days. The diameter of the Sun is 860,000 miles. Find the mass, and the average density of the Sun. (Be sure you show how you got these numbers.)

\[ \frac{GM \cdot m_e}{x^2} = \frac{m_e V^2}{x} \quad V = \frac{2\pi x}{T} \]

\[ \frac{GM_S}{x^2} = \frac{4\pi^2 x}{T^2} \]

\[ \frac{4\pi^2}{G} \left( \frac{92,000,000 \times 5.283 \times 3.28}{6.67 \times 10^{-11} \times (365 \times 24 \times 3600)^2} \right)^3 = 1.93 \times 10^{30} \text{ kg} \]

\[ \rho_{\text{ave}} = \frac{M_{\text{ave}}}{V} = \frac{3M}{4\pi r^2} = \frac{3 \times 1.933 \times 10^{30}}{4\pi \left( \frac{860,000 \times 5.283}{10^3} \right)^3} = 1.39 \times 10^3 \text{ kg/m}^3 \]
An astrophysicist is making a model of a star. He takes the mass as $M_0$ and the radius as $R_0$. He proposes that the density vary as $\rho(r) = \rho_0 (1 - \alpha r^2)$.

(a) Find an expression for $\alpha$ in terms of $M_0$, $R_0$, and $\rho_0$.

$$M_0 = \int_0^R \rho dV = \int_0^R \rho_0 (1 - \alpha r^2) 4\pi r^2 dr$$

$$= 4\pi \rho_0 \left[ \frac{R_0^3}{3} - \alpha \frac{R_0^5}{5} \right]$$

$$\Rightarrow \alpha = \frac{5}{R_0^5} \left( \frac{R_0^3}{3} - \frac{M_0}{4\pi \rho_0} \right)$$

10 pts to point (a)

(b) $\frac{V_a}{2} = \frac{GM}{r^2}$ (gravitational force provides centripetal force)

$$M = \frac{\omega^2 R^3}{G} = \frac{(2\pi)^2 88^3}{G}$$

$$= 4.1 \times 10^{32} \text{ kg}$$

2 significant figures because $T = 88$ days.

5 pts for formula (a) or comparable relation.

10 pts for correct answer with 2 significant figures

8 points if "close" to correct answer

7 points for right order of magnitude (if part (b) correct)

-1 for significant figures

Total points = 10
The radius of the Earth is 4000 miles. Take a model of the Earth in which an inner sphere of radius 2000 miles has an average density of $9.00 \times 10^3 \text{ kg/m}^3$. Find the average density of the outer part of the earth, such that $g$ on the surface remains $9.80 \text{ m/s}^2$. Use only data given on the data sheet, no remembered numbers.

\[ g = \frac{GM_{\text{total}}}{r^2} \]

\[ M_{\text{total}} = \frac{4\pi r_2^3}{3} \cdot \frac{G}{g} = \frac{9.8 \left( 4000 \times \frac{5.28}{3.28} \right)^2}{6.67 \times 10^{-11}} \]

\[ = 6.0917 \times 10^{24} \text{ kg} \quad M_{\text{total}} = M_1 + M_2 \]

\[ \frac{4\pi}{2} \left\{ \rho_1 r_1^3 - \rho_2 r_2^3 + \rho_2 r_1^3 \right\} = 6.0917 \times 10^{24} \]

\[ \rho_2 = \left\{ \frac{3 \times 6.0917 \times 10^{24}}{4\pi} - 9 \times 10^7 \left( 2000 \times \frac{5.28}{3.28} \right)^2 \right\} \]

\[ = \frac{\left( 4000 \times \frac{5.28}{3.28} \right)^2 - \left( 2000 \times \frac{5.28}{3.28} \right)^2}{(4000 \times \frac{5.28}{3.28})^2 - (2000 \times \frac{5.28}{3.28})^2} \]

\[ = 4.94 \times 10^3 \text{ kg/m}^3 \]
Consider a planet where the density varies according to the relationship 
\[ \rho = \rho_0 (1 - \alpha R) \], where \( \rho_0 \) is the density at the center, \( \alpha \) is a constant, and \( R \) is the distance from the center. The radius of the planet is \( R_0 \).

(a) Calculate the mass of the planet.

(b) Calculate the value of \( \vec{g} \) at a point below the surface of the planet, which is a distance \( R_0/3 \) from the center of the planet.

\[ \begin{align*}
M &= \int_{R=0}^{R=R_0} \rho(R) dV = \int_{R=0}^{R=R_0} \rho_0 (1 - \alpha R) 4\pi R^2 dR \\
&= 4\pi \rho_0 \left( \frac{R_0^3}{3} - \frac{\alpha R_0^2}{4} \right) \bigg|_{R=0}^{R=R_0} \\
&= 4\pi \rho_0 \left( \frac{R_0^3}{3} - \frac{\alpha R_0^2}{4} \right)
\end{align*} \]

\[ \begin{align*}
\vec{g} &= \vec{g}(-\hat{r}) = -g \hat{r} \quad (\hat{r} \text{ is unit vector directed radially outward from center of earth})
\end{align*} \]

We know the total contribution of the \( R \) region to the \( \vec{g} \) field at \( R = R_0/3 \) will be zero because \( \rho = \rho(R) \). Therefore \( \vec{g} \) at \( R = R_0/3 \) is due only to the matter inside the radius \( R = R_0/3 \). The mass of this sphere of radius \( R_0/3 \) is

\[ \begin{align*}
M_{R_0/3} &= \frac{4}{3} \pi \rho_0 \left( \frac{(R_0/3)^3}{3} - \frac{\alpha (R_0/3)^2}{4} \right) = \frac{\pi}{81} \rho_0 \left( 4R_0^3 - \alpha R_0^2 \right)
\end{align*} \]

\[ \begin{align*}
\Rightarrow \vec{g} &= \frac{G M_{R_0/3}}{(R_0/3)^2} = \frac{9G \rho_0}{(R_0/3)^2} \left( 4R_0^3 - \alpha R_0^2 \right) = \frac{G \pi \rho_0}{81} \left( 4R_0^3 - \alpha R_0^2 \right)
\end{align*} \]

\[ \begin{align*}
\Rightarrow \vec{g} &= \frac{G \pi \rho_0}{81} (4R_0^3 - \alpha R_0^2) \quad \frac{1}{r^2}
\end{align*} \]
\[ x = 14.2/25 \]
\[ \sqrt{5} = 5.44 \]
\[ n = 331 \]

(b) Assume the planet Mercury orbits the Sun in 90.0 days at a distance of 30,000,000 mi. Use this data to calculate the mass of the sun in kg. It will not be the same as on the data sheet.

\[ \frac{1.022 \times 10^{30}}{\text{kg}} \]

(c) Calculate the magnitude of the electric field at a distance of \(1.00 \times 10^{-10}\) m from a proton. (The charge in the proton is \(e = +1.6 \times 10^{-19}\) C.)

\[ 1.440 \times 10^{16} \text{ N/C} \]

(d) Take the Earth-Moon distance as 240,000 mi. Calculate the gravitational force in Newtons between the Earth and the Moon.

\[ 1.765 \times 10^{26} \text{ N} \]

(e) A fine wire 12.0 m long has a total charge of \(+4.25 \times 10^{-8}\) C. Calculate the electric field and direction 1.00 mm away from the center of the wire at a point halfway between the ends.

\[ 0.37 \times 10^4 \text{ N/C} \] away from wire
a) \((1 + x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \frac{n(n-1)(n-2)}{3!} x^3 + \ldots\)

Here, \(n = -\frac{9}{2}\),

then \(a_3 = \frac{(-\frac{9}{2})(-\frac{9}{2}-1)(-\frac{9}{2}-2)}{3!} = -\frac{9 \cdot 11 \cdot 13}{8 \cdot 6} = -26.81\)

Common error:

The coefficient of \(x^3\) is not the first 4 terms

Grading:
- 2 points for number and (+) wrong sign
- 5 points for number and correct sign

b) \(T = 90.0 \text{ days} = 7.776 \times 10^6 \text{ sec.}\)

\(r = 30,000,000 \text{ miles} = 4.829 \times 10^{10} \text{ m}\)

\(F = G \frac{M m}{r^2} = m \left(\frac{2\pi^2}{T^2}\right)^2\)

\(\Rightarrow M = \frac{4\pi^2 r^3}{9G T^2} = 1.1023 \times 10^{20} \text{ kg}\)

c) \(E = k \frac{8}{r^2} = 1.44 \times 10^{11} \text{ N/m}\)

d) \(F = G \frac{M m}{r^2} = 1.9646 \times 10^{20} \text{ N}\)

e) \(L = 12.0 \text{ m}\)

\(r = 10^{-3} \text{ m}\)
Note that real, so "intuitively" you can see Gauss's Law is ok. In that case

\[ E_{2rr} = \frac{Q}{\epsilon_0} \]

\[ E = \frac{Q}{2\pi \epsilon_0 r} = 6,369 \times 10^{-8} \text{ N/C} \]

How good is this? Do it exactly.

\[ \Theta \bar{dE} \]

\[ (dE) = \frac{k \bar{d} \theta}{x^2 + r^2} \]

By symmetry, we can see that the components parallel to the wire cancel.

Then:

\[ E = \int dE_{\perp} = \int dE_{\perp} = \int \frac{k \bar{d} \theta}{x^2 + r^2} \]

\[ \int \frac{k \bar{d} \theta}{x^2 + r^2} \]

let \( d\theta = 2 \bar{d}x \), \( \bar{x} = \frac{Q}{L} \)

Then:

\[ E = k \lambda \int \frac{d\bar{x}}{(\frac{x}{2})^2 + r^2} = \text{Credit (5 pts)} \]

was given for this

\[ = \frac{k \lambda}{r} \left[ \frac{L}{(\frac{L}{2})^2 + r^2} \right] \]

\[ = \frac{2kQ}{rL} \left[ \frac{1}{1 + (\frac{2r}{L})^2} \right] \]

\[ = \frac{\bar{Q}}{2\pi \epsilon_0 r} \left[ 1 - \frac{1}{2} \left( \frac{2r}{L} \right)^2 + \ldots \right] \]

\[ \approx \frac{\bar{Q}}{2\pi \epsilon_0 r} \approx \text{Gauss's Law result} \]

Relative error \( \approx \frac{1}{2} \left( \frac{2r}{L} \right)^2 = \frac{1}{2} \left( \frac{2 \times 10^{-3}}{12} \right)^2 = 1.39 \times 10^{-8} \)
A satellite of mass 150 kg is in a circular orbit 8000 mi above the Earth's surface.

(a) Calculate the velocity of the satellite in this orbit.
(b) Calculate the total energy of the satellite in this orbit.
(c) Calculate the minimum amount by which the satellite's velocity must be increased in order that it escapes from the Earth. Ignore the Sun and all other planets.

\[ a) \quad \frac{GM_\oplus m_s}{(R_\oplus + h)^2} = \frac{m_s v^2}{(R_\oplus + h)} \quad \Rightarrow \quad v = \sqrt{\frac{G M_\oplus}{R_\oplus + h}} = \sqrt{\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{1.38 \times 10^{10} + 8000 \times 5280}} = 4.55 \times 10^3 \text{ (m/s)} \]

or \[ T = \sqrt{\frac{\pi^2 r^3}{G M_\oplus}} \quad \Rightarrow \quad v = \frac{2\pi r}{T} \quad \text{ where } \quad r = R_\oplus + h = 1.926 \times 10^8 \text{ (m)} \]

(b) \[ E = K + U = -\frac{G M_\oplus m_s}{2h} = -1.55 \times 10^9 \text{ (J)} \]

or \[ E = K + U = \frac{1}{2} m_s v^2 - \frac{G M_\oplus m_s}{r} \]

(c) \[ \text{V}_{\text{esc.}} = \sqrt{\frac{2GM_\oplus}{r}} = 6.44 \times 10^3 \text{ (m/s)} \]

\[ \Delta V = \text{V}_{\text{esc.}} - v = 1.89 \times 10^3 \text{ (m/s)} \]

or \[ \Delta E + \frac{1}{2} m_s v^2 - \frac{G M_\oplus m_s}{r} = 0 \quad \text{(The energy of sat. at \( \infty \))} \]

\[ \Delta E = \frac{1}{2} m_s (v + \Delta v)^2 - \frac{1}{2} m_s v^2 \]

\[ v + \Delta v = \sqrt{\frac{2GM_\oplus}{r}} \]

\[ \Delta v = \sqrt{\frac{2GM_\oplus}{r}} - v = 1.89 \times 10^3 \text{ (m/s)} \]
FIRST EXAM

Name (Print) _______________________________ Name (Signed) _______________________________ Average = 17.6

Discussion Instructor (Circle One): Cady McAllister Molina Stone

Discussion Section #: _______________________________

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front panel.

(a) Calculate the speed (in m/s) of a satellite in a circular orbit 1000 mi above the surface of the earth. 
\[ r = 1000 \text{ mi} + R_E = 7.99 \times 10^6 \text{ (mi)} \]
\[ v = \sqrt{\frac{GM_E}{r}} = \sqrt{\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{7.99 \times 10^6}} = 7.07 \times 10^3 \text{ (m/s)} \]

(b) Calculate the total energy of the satellite in (a) if its mass is 1500 kg.
\[ E_T = -\frac{GMmE}{r} = \frac{1}{2}mv^2 = \frac{10}{3} = 3.33 \times 10^3 \text{ (J)} \]

(c) Calculate the gravitational force between two elephants (assumed to be spheres) if there is 5.00 m between their centers. Take the mass of each elephant as 4000 kg.
\[ F = \frac{Gm_1m_2}{r^2} = \frac{6.67 \times 10^{-11} (4210^3)^2}{6^2} = 2.96 \times 10^5 \text{ (N)} \]

(d) Assume that a black hole has a total mass 3.00 times the mass of the sun. Calculate the speed of a small object in a circular orbit 1000 miles from the center of the black hole. 
\[ r = 1000 \text{ mi} = 1.61 \times 10^6 \text{ (mi)} \]
\[ M = 3M_S \]
\[ v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{3 \times 1.98 \times 10^{30} \times 6.67 \times 10^{-11}}{1.61 \times 10^6}} = 1.57 \times 10^7 \text{ (m/s)} \]

(e) Calculate the magnitude of the electric force between two electrons a distance of $1.00 \times 10^{-11}$ m apart.
\[ F = \frac{k_e^2}{r^2} = \frac{9 \times 10^9 \cdot (1.60 \times 10^{-19})^2}{(10^{-11})^2} = 7.30 \times 10^{-2} \text{ (N)} \]
The planet Pluto has a moon, Charon, whose orbital period is 6 days. Assume that the mass of Charon is small compared with the mass of Pluto (probably not true). Assume that the mass of Pluto is \(0.10 M_E\).

(a) Find the radius of Charon's orbit.

(b) If the radius of Pluto is 2000 km, find the escape speed from its surface.

(c) Assume that Pluto is in a circular orbit of radius \(5.90 \times 10^9\) km about the sun. Calculate the orbital period of Pluto about the sun.

\[ \begin{align*}
\mathbf{T} &= (60\text{day} \cdot 24\text{hr day}^{-1} \cdot 3600\text{sec hr}^{-1}) = 5.184 \times 10^5 \text{s} \\
M_P &= 5.98 \times 10^{23}\text{ kg} \\
\mathbf{r} &= \left(\frac{G M_P T^2}{4 \pi^2}\right)^{1/3} = \left(\frac{6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2} \cdot 5.98 \times 10^{23}\text{kg} \cdot (5.184 \times 10^5)^2}{4 \pi^2}\right)^{1/3} \\
\mathbf{r} &= 6.48 \times 10^7\text{ m} \\

\mathbf{v}_{\text{esc}} &= \sqrt{\frac{2GM_P}{R_P}} \\
\mathbf{v}_{\text{esc}} &= \sqrt{\frac{2 \cdot 6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2} \cdot 5.98 \times 10^{23}\text{kg}}{2 \times 10^6\text{m}}} \\
\mathbf{v}_{\text{esc}} &= 6.32 \times 10^3\text{ m/s} \\

\mathbf{T} &= \sqrt{\frac{4\pi^2}{GM_\odot} r^3} = \sqrt{\frac{4\pi^2 (5.90 \times 10^{12}\text{m})^3}{6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2} \cdot 1.99 \times 10^{30}\text{kg}}} \\
\mathbf{T} &= 1.82 \times 10^9\text{s} \\
\text{or} & \quad 248\text{ yrs}
\end{align*} \]
1. First Midterm #2 \[ N = 285 \quad \bar{x} = 51.6 \]

Common Mistakes

10 a) Inverting numerator & denominator. -5
   Didn't: \( \pi^2 \), \( \Gamma^2 \), \( \sqrt{\pi} \) - 3
   Right #’s wrong results - 3

5 b) Didn't: convert km \( \rightarrow \) m, \( \sqrt{\pi} \),
   Forgot \( \times 10^3 \); other wrong exponent
   Used radius of Charon or other - 3
   Wrong \( r \)

0 c) Didn't: convert km \( \rightarrow \) m, \( \pi^2 \), \( \sqrt{\pi} \), \( r^3 \) - 3
   Used Mpluto instead of Msmn - 5
FIRST MIDTERM

(a) If a satellite circles the Earth in a circular orbit 500 km above the surface of the Earth in exactly 90 min, what is the mass of the Earth (not quite the same as on the data sheet)?

\[ M = \frac{4\pi^2 Y^3}{T^2} \]
\[ Y = R_e + h = 0.88 \times 10^3 \text{ km} \]
\[ T = 90 \times 60 = 5400 \text{ sec} \]

(b) Calculate the total energy of the Earth in its orbit around the Sun. Assume the radius of the orbit is \(1.48 \times 10^8 \text{ km} \).

\[ T_{\text{total}} = T_k + T_F = -\frac{1}{2} \frac{G M m}{Y} \\
= - \frac{1}{2} \times 6.67 \times 10^{-11} \times \frac{1.99 \times 10^{30} \times 5.98 \times 10^{24}}{1.48 \times 10^{11}} \]

(c) Calculate the magnitude of the electric force between two electrons that are \(0.2 \times 10^{-10} \text{ m} \) apart.

\[ F = k \frac{e^2}{r^2} = 9.00 \times 10^9 \times \frac{(1.6 \times 10^{-19})^2}{(0.2 \times 10^{-10})^2} \]

(d) Using the binomial expansion on \((1 + x^{3})^{4/3} \) where \(x \ll 1\), calculate the coefficient of the term in \(x^6\).

\[ (1 + x^3)^{4/3} = 1 + (\frac{4}{3})x^3 + \frac{1}{2} \cdot \frac{4}{3} \cdot (\frac{4}{3} - 1)x^6 + \frac{1}{3} \cdot (\frac{4}{3})(\frac{1}{3} - 1)(\frac{1}{3} - 2)x^9 + \ldots \]

(e) If a cube has a total charge inside of \(3.20 \text{ nC} \), calculate the total electric flux, with units, crossing the faces of the cube.

\[ 3.62 \times 10^2 \text{ N m}^2 / \text{C} \]

\[ \Phi = \frac{Q}{\varepsilon_0} = \frac{3.20 \times 10^{-9}}{8.85 \times 10^{-12}} = 3.62 \times 10^3 \]
FIRST MIDTERM

Name (print) \textbf{Mark Reeve} \hspace{2cm} Name (signed) \textbf{Mark Reeve}

Discussion Instructor (circle one): Baselgia  Morrill  Reeve  Stoops  Zhang

Discussion Section #

SHOW ALL WORK!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

Assume that a geologist wishes to make a model of the Earth where the density varies as 
\( \rho(R) = \rho_o (1 - \alpha R^2) \). Assume that the radius of the Earth (\( R_E \)) and the mass of the Earth (\( M_E \)) are as given on the data sheet. Use \( \rho_o = 7.50 \times 10^3 \text{ kg/m}^3 \).

(a) Find the value of \( \alpha \).
(b) Calculate the value of \( g \) at a point \( R = R_E/2 \).

(If you can't do (a), leave \( \alpha \) in (b) as a symbol.)

a) \( 15 \) points \( \quad M_E = \int \rho \, dV \quad \quad \quad \quad \quad \quad \quad dV = 4\pi r^2 \, dr \)
\[ \int_0^{R_E} \rho_o (1 - \alpha r^2) \cdot 4\pi r^2 \, dr = 4\pi \rho_o \left[ \frac{R_E^3}{3} - \frac{\alpha R_E^5}{5} \right] = M_E \]

Solving for \( \alpha = \frac{5}{R_E^2} \left( 1 - \frac{M_E}{\frac{4}{3} \rho_o R_E^3} \right) \)
\[ \alpha = 1.043 \times 10^{-14} \text{ m}^{-2} \]

+8 points for getting the correct integral
+5 points for correctly solving for \( \alpha \)
+2 points for correctly solving the numbers accurately
-1 for more or less than 3 s.f.
-1 for incorrect or missing units (extremely common)

b) \( 10 \) points \( \quad g(r) = \frac{GM}{r^2} \) where \( M \) lies within a sphere of radius \( r \).
\[ g(\frac{R_E}{2}) = \frac{G}{(\frac{R_E}{2})^2} \int_0^{\frac{R_E}{2}} \rho \, dV = \frac{4\pi G}{R_E^2} \cdot 4\pi \rho_o \int_0^{\frac{R_E}{2}} (1 - \alpha r^2) r^2 \, dr \]
\[ = 16\pi G \rho_o \left[ \frac{r^3}{3} - \frac{\alpha r^5}{5} \right]_0 = 16\pi G \rho_o \frac{R_E^3}{5} \left[ \frac{2}{5} - \frac{\alpha R_E^5}{5} \right] \]
\[ = g(\frac{R_E}{2}) = 6.24 \text{ m/sec}^2 \]

+6 pts for the integral
+2 pts for algebra
+2 pts for numbers

Many people failed to integrate to find the mass in spherical shell parts since the density varies with the radius, you must integrate.
FIRST MIDTERM

Average = 19.3

Name (print) ____________________________ Name (signed) ______  
N = 25B

Discussion Instructor (circle one): Davis DeTienne Named Molina Paul Zhang

Discussion Section # _________

1 Unit

1 Unit

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

(a) If \( g \) on the surface of Mars is \( 3.70 \text{ m/s}^2 \) and the mass of Mars is \( 6.00 \times 10^{23} \text{ kg} \), calculate the radius of Mars.

\[
\frac{G}{R^2} = 3.70 \implies R = \left( \frac{GM}{3} \right)^{1/2} = 3.29 \times 10^6 \text{ (m)}
\]

(b) Calculate the magnitude of the electric force between an electron and a proton (the hydrogen nucleus) if they are \( 0.75 \times 10^{-10} \text{ m} \) apart. The charge of the proton is positive and is equal in magnitude to the charge of the electron. (Numerical answer.)

\[
F = \frac{k e^2}{r^2} = 4.11 \times 10^{-8} \text{ (N)}
\]

(c) If the Moon has an average density of \( 3340 \text{ kg/m}^3 \) and a radius of 1738 km, what is the mass of the Moon?

\[
M = \frac{4}{3} \pi R^3 \rho = 7.34 \times 10^{22} \text{ (kg)}
\]

(d) Calculate the electric field 2.00 m from the surface of a uniformly charged non-conducting sphere, whose radius is 3.00 m and whose total charge is \( 5.75 \times 10^{-3} \text{ C} \). (Numerical answer.)

\[
E = \frac{kQ}{r^2} = \frac{(9 \times 10^9 \times 5.75 \times 10^{-3})}{2^2} = 7.04 \times 10^6 \text{ (N/C)}
\]

(e) Use the binomial expansion to calculate the complete term in \( a^4 \) for

\[
\left( 1 - \frac{a^2}{x^2} \right)^{-3/2}
\]

\[
\frac{35}{8x^4}
\]
SHOW ALL WORK!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES
Use the conversion constants and data given on the front page.

Captain Kirk and the Enterprise are in orbit about a planet whose radius they measure as 6250 km. When they are in a circular orbit a distance of 24,000 km above the surface of the planet, they measure the period of that orbit as 3.34 hrs. Calculate the density of the planet.

\[ r = 8.25 \times 10^4 \text{m} \]
\[ T = 3.34 \text{ hrs} \cdot \frac{60 \text{ mins}}{1 \text{ hr}} = 1.20 \text{ hrs} \]

\[ R = 2.4 \times 10^4 \text{ m} + 8.25 \times 10^4 \text{ m} = 3.225 \times 10^5 \text{ m} \]

The force on the spaceship due to gravity is given by

\[ F = \frac{GMm}{R^2} \]

By Newton's second law,

\[ F = \frac{GMm}{R^2} = ma = \frac{mV^2}{R} \]

\[ \frac{GM}{R^2} = \frac{V^2}{R} \]

\[ \frac{GM}{R^2} = \rho \frac{4}{3} \pi R^3 \]

\[ \rho = \frac{3 \pi R^3}{G \cdot \frac{V^2}{R}} = 5.84 \times 10^9 \text{ kg/m}^3 \]
FIRST MIDTERM

Name (print)  Zhang Tian

Discussion Instructor (circle one): Davis DeTienes Hamed Motian Paul Zhang

Discussion Section #

SHOW ALL WORK!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

The density of a star is approximated by \( \rho = \rho_0 (1 - \alpha R^2) \), where \( \alpha \) and \( \rho_0 \) are constants, and the density goes to zero at the surface, \( R = R_0 \). The radius of the star is \( 1.20 \times 10^6 \) km, and its total mass is \( 8.00 \times 10^{30} \) kg.

(a) Calculate \( \alpha \).
(b) Determine \( \rho_0 \).
(c) Find the value of \( g \) at point \( R_0 \) (inside the star).

\[ R = R_0, \quad \rho = 0 \Rightarrow \alpha = \frac{1}{R_0^2} = 6.94 \times 10^{-19} \text{ m}^{-2} \]  

\[ M = \int_{R}^{R_0} 4\pi R^2 \rho \, dR = \int_{R}^{R_0} \rho_0 (1 - \alpha R^2) 4\pi R^2 \, dR = 4\pi \rho_0 R_0^3 \left[ \frac{1}{3} R_0^3 - \frac{1}{5} \left( \frac{R_0}{R} \right)^3 \right] R_0 \\
= 4\pi \rho_0 \left[ \frac{1}{3} R_0^3 - \frac{1}{5} R_0^3 \right] R_0^3 = \frac{8\pi}{15} \rho_0 R_0^3 \]

\[ \rho_0 = \frac{15M}{8\pi R_0^2} = 2.17 \times 10^3 \text{ kg/m}^3 \]  

\[ g = G \frac{M}{(R_0)^2} = 9.8 \times \frac{R_0^3}{M} = 12\pi \rho_0 R_0 \left( \frac{1}{27} - \frac{1}{81} \right) G = \frac{12}{27} \pi \rho_0 R_0 \frac{14}{15} G \\
= \frac{12}{27} \times \frac{15M}{8\pi R_0^2} \frac{14}{15} R_0 G = \frac{21}{27} MG \frac{1}{R_0^2} = 2.88 \times 10^8 \text{ m/s}^2 \]
The Starship Enterprise is in a circular orbit about an unknown planet. The kinetic energy of the Enterprise is \( 1.60 \times 10^{19} \text{ J} \) and its mass is \( 4.00 \times 10^7 \text{ kg} \). The orbit is \( 14.0 \times 10^6 \text{ m} \) above the surface of the planet, whose radius is \( 7.00 \times 10^6 \text{ m} \).

(a) Calculate the potential energy (numerical value) of the Enterprise in this orbit. Ignore influences other than this planet.

(b) Determine the total energy (numerical value) of the Enterprise.

(c) Find the average mass density of this planet.

\[ KE = \frac{1}{2} M_E V^2 \]

\[ PE = \frac{-GM_P M_E}{r} \]

\[ E_{\text{total}} = KE + PE = 1 \times 10^{19} + (-2.00 \times 10^{19}) \]

\[ \rho = \frac{M_P}{V_P} = \frac{M_P}{\frac{4}{3} \pi r_P^3} = \frac{(1.5742 \times 10^{29})}{4 \pi (7 \times 10^6)^3} \]

\[ = 1.10 \times 10^8 \text{ kg/m}^3 \]
A geophysicist decides to model the earth as having a mass density that diminishes linearly with the distance from the center using $\rho(r) = \rho_0 - \alpha r$ where $\rho_0$ and $\alpha$ are constants and $r$ is the distance from the center of the earth.

(a) If he chooses $\rho_0 = 1.00 \times 10^4$ kg/m$^3$ find the magnitude and units of $\alpha$ that gives the correct mass for the earth.

(b) Using the above form for the mass density, calculate the magnitude of the effective "g" at $R = 4.00 \times 10^3$ km from the center of the earth.

$$M = \int_0^R \rho(r) \, dV$$

$$= \int_0^R (\rho_0 - \alpha r) \cdot r^2 \, dr$$

$(a)$

$$M_E = 4\pi \int_0^{R_E} (\rho_0 - \alpha r) r^2 \, dr = \frac{4}{3} \pi R_E^3 \rho_0 - \pi \alpha R_E^4$$

$$\Rightarrow \quad \alpha = \frac{4}{3} \frac{\rho_0}{R_E} - \frac{M_E}{\pi R_E^4} \quad , \quad R_E = 6.38 \times 10^6 \, \text{m}, \, M_E = 5.98 \times 10^{24} \, \text{kg}$$

$$\alpha = \frac{4 \times 10^4}{3 \times 6.58 \times 10^6} - \frac{5.98 \times 10^{24}}{3.14 \times (6.38 \times 10^6)^2} = 9.41 \times 10^{-4} \, \left[ \frac{\text{kg}}{\text{m}^2} \right]$$

$(b)$

$$g_r = G \frac{M_E}{R^2} = \frac{G}{R^2} \int_0^R (\rho_0 - \alpha r) r^2 \, dr =$$

$$= 4\pi \frac{G}{R^2} \left( \frac{\rho_0}{3} R^3 - \frac{\alpha}{4} R^4 \right) = \pi G \left( \frac{4}{3} \rho_0 R^3 - \alpha R^4 \right)$$

$$g_r = 3.14 \times 6.67 \times 10^{-11} \left[ \frac{4}{3} \times 4 \times 10^6 \times 10^4 - 9.41 \times 10^{-4} \times (4.10^6)^2 \right] =$$

$$= 8.02 \, \frac{\text{m}}{\text{s}^2}$$
FIRST MIDTERM

Name (print) ___________ Name (signed) ___________

Discussion Instructor (circle): Brown Chakhbaziian Condella Portnoi Zhukov

Discussion Section # ______

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES:
Use the conversion constants and data given on the front page.

(a) For the expression,

\[
\frac{1}{(1 - x)^{5/2}} = 1 + \frac{5}{2}(-x) + \frac{5}{2} \cdot \frac{3}{2} \left(\frac{x}{2} \right)^2 + \frac{5}{2} \cdot \frac{3}{2} \cdot \frac{1}{2} \left(\frac{x}{2} \right)^3 = 
\]

calculate the \( x^3 \) term in the binomial expansion.

\[
= 1 - \frac{5}{2}x + \frac{15}{8}x^2 - \frac{15}{16}x^3.
\]

(b) An electron is accelerated from rest in an electric field of \( 2.35 \times 10^8 \) N/C. Calculate its speed after it has traveled 7.00 cm.

\[
\frac{V_e^2}{2a} = s \quad a = \frac{F}{ma} = \frac{eF}{m} \quad V = \frac{2eE}{m} = 2.40 \times 10^6 \text{ m/s}
\]

(c) If the radius of the earth were doubled and the mass of the earth increased by a factor of 10.0, what is the weight of a student who now weighs 1200 N?

\[
G^1 = 2R^1, M^1 = 10M \quad W = \frac{G m M^1}{R^1}, W^1 = 6 \frac{m M^1}{R^1} = \frac{G m M}{4 R} = \frac{12}{4} W = 3000
\]

(d) Calculate the magnitude of the electric force between an electron and the nucleus of a hydrogen atom. The nuclear charge is equal to the charge of the electron. Take the distance between them as \( 1.00 \times 10^{-10} \) m.

\[
F = \frac{e^2}{\varepsilon_0} = 2.30 \times 10^{-8} \text{ N}
\]

(e) Calculate the speed (in m/s) of a satellite in a circular orbit 2000 km above the surface of the earth.

\[
\frac{m v^2}{R} = \frac{G m M}{R^2}, v = \sqrt{\frac{G m}{R + R_e}} = 6.30 \times 10^3 \text{ m/s}
\]
FIRST MIDTERM

Name (print) Solutions Name (signed) Portnoi

Discussion Instructor (circle): Brown Chakhbazian Condella Portnoi Zhukov

Discussion Section # __________

SHOW ALL WORK!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

The planet Mercury orbits the sun in a period of 160.0 days at a distance of 40,000,000 miles (number are not quite the true value).

(a) Calculate the mass of the sun in kg using the above numbers.

(b) If the radius of the sun is 500,000 miles, find its mass density in kg/m^3 using the mass obtained in part (a).

\( T = 160 \text{ days} = 160 \text{ days} \cdot \frac{24 \text{ hours}}{\text{day}} \cdot \frac{3600 \text{ sec}}{\text{hour}} \approx 8.64 \times 10^5 \text{ s} \)

\( R = 4.10^7 \text{ miles} = 4.10^7 \text{ miles} \cdot \frac{5280 \text{ ft}}{\text{mile}} \cdot \frac{1 \text{ m}}{3.28 \text{ ft}} = 6.44 \times 10^{10} \text{ m} \)

\[ T^2 = \left(\frac{4\pi^2}{GM}\right) R^3 \Rightarrow M = \frac{4\pi^2}{G} \frac{R^3}{T^2} = 2.12 \times 10^3 \text{ kg} \]

\[ \rho = \frac{M}{\frac{4}{3} \pi r^3} \Rightarrow r = 5 \times 10^6 \text{ miles} \approx 8.05 \times 10^8 \text{ m} \Rightarrow \]

\[ \rho = 9.69 \times 10^2 \frac{\text{kg}}{\text{m}^3} \]
First Midterm

Name (print) ____________________________  Name (signed) ____________________________

Discussion Instructor (circle): Brown Chakrabarri Condella Portnoi Zhukov

Discussion Section # ______

Show All Work!!!!
Report All Numbers to Three Significant Figures!
Use the conversion constants and data given on the front page.

For the following question, use the data from the data sheet and an earth-sun distance of 93,000,000 miles between their centers.

(a) Find the total energy (kinetic plus potential) of the earth in its orbit (assume a circular orbit).
(b) Calculate the total speed needed for the earth to leave its orbit and escape from the sun. That is, find the velocity at its present distance from the sun needed to escape.

\[ E = T + V = \frac{mv^2}{2} - \frac{GMmM}{RE-s} \]

\[ mV^2 = \frac{GMmM}{R^2} \]

\[ v_m = \frac{GM}{RE-s} \]

\[ \Rightarrow E = -\frac{GMmM}{2RE-s} = -2.64 \times 10^{33} \text{ J} \]

\[ \frac{-6mM}{RS-E} + \frac{1}{2} mV^2 = 0 \]

\[ \Rightarrow v^2 = \frac{2GMs}{RS-E} = \frac{-4E}{m} \]

\[ v = 4.20 \times 10^4 \text{ m/s} \]
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

(a) A 2000 kg car going 60 km/hr hits a stopped car (also 2000 kg). Ignore friction. What is the total momentum after this collision? 
\[ m = 2000 \text{ kg} \]
\[ v = 60 \text{ km/hr} \]
\[ = 60 \times \frac{1000 \text{ m}}{3600 \text{ s}} = 16.67 \text{ m/s} \]
\[ \vec{p} = m\vec{v} = \text{Const.} \]
\[ p = mv = 2000 \times 60 = 120000 \text{ kg \cdot km/hr} \]
\[ (or \, p = 2000 \times 16.67 = 33333.3 \text{ kg \cdot m/s}) \]

(b) Find the center of mass of the system shown. [Hint: Show your coordinate system and choose it to take advantage of symmetry.]
For example, as shown in Figure,
\[ x_{cm} = \frac{m(-\frac{a}{3}) + 2m(\frac{a}{3}) + m \cdot 0}{7m} = \frac{2a}{7} = 0.286a \]
\[ y_{cm} = \frac{m \cdot 0 + 5m \cdot 0 + m \cdot \sqrt{a^2 - \frac{a^2}{4}}}{7m} = \frac{a^2}{4} \]
\[ a = 0.124a \]

(c) Using the constants given on the data sheet, find the average density of the earth.
\[ \rho = \frac{m}{V} = \frac{m}{\frac{4}{3} \pi R^3} = \frac{5.98 \times 10^{27}}{\frac{4}{3} \pi \times 3.9 \times (6.38 \times 10^6)^3} = 5.5 \times 10^{3} \text{ kg/m}^3 \]

(d) Find the ratio of the orbital speed for two satellites in circular orbits 7000 km above the center of the moon and above the center of the earth.
\[ \frac{G \frac{M}{R^2}}{m} = \frac{G \frac{M}{R^2}}{m} \]
\[ \frac{m}{M} \frac{V_e^2}{V_m^2} \rightarrow \frac{M_e}{M_m} = \frac{V_e^2}{V_m^2} \rightarrow \frac{V_e}{V_m} = \sqrt{\frac{M_e}{M_m}} = \sqrt{\frac{5.98 \times 10^{24}}{7.35 \times 10^{22}}} = 0.2 \]
\[ (or \, \frac{V_e}{V_m} = 0.111) \]

(e) A planet orbits the sun (circular orbit) in 75 years. In units of the distance between the sun and the earth [R(sun-earth) = 1 A.U.], how far away is the planet from the sun?
\[ T = 75 \text{ years} = 7.5 \times 365 \times 24 \times 3600 = 2.365 \times 10^9 \text{ s} \]
\[ \frac{GMm}{R^2} = \frac{m}{r^2} \]
\[ \frac{3}{4 \pi G \frac{m}{2}} \frac{T^2}{R^2} = \frac{6.67 \times 10^{-11} \times 1.99 \times 10^3 \times (2.365 \times 10^8)^2}{4 \times 3.14^2} \]
\[ \frac{3}{4 \pi \frac{m}{2}} \frac{T^2}{R^2} = \frac{2.66 \times 10^{12} \text{ m}}{1.5 \times 10^{10}} = 17.7 \text{ A.U.} \]
Problem # 3

a) Since we have the time required to complete one complete revolution for the satellite 1 is given by

\[ T_1 = \sqrt{\frac{4\pi^2 (r_1^3)}{GM_E}} \]

where \( r_1 = h_1 + R_E \)

\[ M_E \] mass of earth

\[ h_1 = 230 \text{ Km} = 230 \times 10^3 \text{ m}, \ R_E = \text{radius of Earth} \]

\[ T_1 = \sqrt{\frac{4\pi^2 \left(230 \times 10^3 + 6.38 \times 10^8\right)^3}{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}} \]

\[ = 5.33 \times 10^3 \text{ sec} \approx 1.48 \text{ h} \]

Since sat 2 completes \( \frac{1}{2} \) revolution in the time sat 1 completes one complete revolution

\[ T_2 = 2T_1 \]

i.e., The shortest time after \( t = 0 \) required for the two satellites to be on opposite sides of the earth (as shown in fig. 2) is 1.48 h.
3b) The answer to this part is exactly like in figure 2, since one of the satellites will complete its own revolution while the other \( \frac{1}{2} \) revolution of its own orbit.

3c) In case of \( m_1 = m_2 = m \), the total energy of sat in its own orbit is given by

\[
E_{\text{tot}} = KE + PE = \frac{1}{2}mv^2 + \frac{-GMmE}{r}
\]

\[
= \frac{1}{2}mv^2 - \frac{GMmE}{r} = \frac{GMmE}{2r} - \frac{GMmE}{r}
\]

\[
\therefore E_{\text{tot}} = -\frac{GMmE}{2r} = \frac{1}{2}PE = -\frac{\text{const}}{r}
\]

Sat 1 has lower altitude than Sat 2. Then Sat 1 has smaller PE.

\[
E_{t1} = -\frac{\text{const}}{h_1 + RE}, \quad E_{t2} = -\frac{\text{const}}{h_2 + RE}
\]

\[
h_1 < h_2 \quad \text{const is the same = \frac{GMmE}{2}}
\]

\[
\therefore \frac{1}{h_1 + RE} > \frac{1}{h_2 + RE} \Rightarrow -\frac{1}{h_1 + RE} < -\frac{1}{h_2 + RE}
\]

\[
\Rightarrow -\frac{\text{const}}{h_1 + RE} < -\frac{\text{const}}{h_2 + RE} \Rightarrow E_{t2} > E_{t1}
\]
1. Calculating \( T \) from the relation
\[
T = \sqrt{\frac{4\pi^2 r^3}{GM}}
\]
Using \( r \) in km, and the time will be wrong. (2 pts)

2. Assuming the total energy (with +ve magnitude) the more +ve value is greater (3 pts)

3. sat of more KE is also of more total energy although it is of lower altitude (lower PE) (3 pts)

4. drawing the positions of the satellites, they assume that the difference between the periods is \( \frac{3}{4} \) period, i.e., if one is at \( \frac{1}{4} \) the other is at (5 pts)

\[
\Delta \text{ periods} = \frac{1}{2} \text{ period}
\]
5) Some forget $\pi^2$ in the relation:

$$T = \sqrt{\frac{4\pi^2 r E}{GM_e}}$$  \hspace{1cm} (1 pt)

6) Some use:

$$T = \sqrt{\frac{4\pi^2 r}{GM_e}} \quad \text{where} \quad r = \text{altitude}$$

\hspace{1cm} \text{without adding } R_E  \hspace{1cm} (3 pts)
Problem 1

(a) \[ \text{K.E.}_i + P.E._i = \text{K.E.}_f + P.E._{f} \]
\[ \frac{1}{2} m v_i^2 + \left( -\frac{G M m}{R_e} \right) = \frac{1}{2} m v_f^2 + \left( -\frac{G M m}{R_e + h} \right) \]
\[ \frac{1}{2} v_f^2 - \frac{G M}{R_e} = - \frac{G M}{R_e + h} \]
\[ \Rightarrow h = \frac{v_f^2 R_e^2}{2 G M - v_f^2 R_e} \]
\[ v_f = 2.05 \times 10^3 \text{ m/s} \]
\[ R_e = 6.38 \times 10^6 \text{ m} \]
\[ G = 6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \]
\[ M = 5.98 \times 10^{24} \text{ kg} \]

\[ \Rightarrow h = 221891 \text{ m} \approx 222 \text{ km} \]

(b) \[ X_{CM} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3}{m_1 + m_2 + m_3} = \frac{0.5 + 2.0 + 0.2}{m + m + 2m} = \frac{e}{2} \]
\[ Y_{CM} = \frac{y_1 m_1 + y_2 m_2 + y_3 m_3}{m_1 + m_2 + m_3} = \frac{0.5 + 0.0 + 1.2}{m + m + 2m} = \frac{e}{2} \]

(c) \[ \vec{p}_i = \vec{p}_f \Rightarrow m_1 \vec{v}_1 + m_2 \vec{v}_2 = (m_1 + m_2) \vec{v} \]
\[ \vec{v} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2} \]

\[ v = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = 0.427 \frac{m}{s} \]

(d) Before: \[ K_i = K_{total} - K_{CM} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 - \frac{1}{2} (m_1 + m_2) v^2 \]
\[ K_i = 0.0107 \]

After \[ K_f = 0 \]

(e) \[ \vec{p}_i = \vec{p}_f \Rightarrow \vec{p}_i = \vec{p}_1 + \vec{p}_2 \]
\[ \begin{align*}
Ox: \quad v_{ix} &= m_1 v_{ix} \cos \theta + m_2 v_{ix} \cos \theta \quad \text{Oy:} \quad v_{iy} &= m_1 v_{iy} \sin \theta - m_2 v_{iy} \sin \theta.
\end{align*} \]

Only for: \( v_{i1} = v_{i2} \) and \( \theta = 30^\circ \):
\[ v_{i2} = \frac{v_1}{\sqrt{3}} = g.24 \frac{m}{s^2} \]
Problem 1*

(1) \[ K.E. + P.E. = K.E. + P.E. \]
\[ \frac{1}{2} m v^2 + \left( -\frac{GMm}{Re} \right) = \frac{1}{2} m v'_{O}^2 + \left( -\frac{GMm}{Re + h} \right) \]

\[ \frac{1}{2} v^2 - \frac{GM}{Re} = -\frac{GM}{Re + h} \quad \Rightarrow \quad h = \frac{v^2 Re}{2GM - v^2 Re} \]

\[ v = 4.05 \times 10^3 \text{m/s} \]
\[ Re = 6.38 \times 10^6 \text{m} \]
\[ G = 6.67 \times 10^{-11} \text{N} \cdot \text{m}^2 / \text{kg}^2 \]
\[ M = 5.98 \times 10^{24} \text{kg} \]

\[ \Rightarrow h = 963308 \text{m} = 963 \text{Km} \]

(2) \[
\begin{align*}
X_{cm} &= \frac{X_1 m_1 + X_2 m_2 + X_3 m_3}{m_1 + m_2 + m_3} = \frac{0.1 m + 3.0 m + 0.3 m}{m + m + 3m} = \frac{3}{5} l \\
Y_{cm} &= \frac{Y_1 m_1 + Y_2 m_2 + Y_3 m_3}{m_1 + m_2 + m_3} = \frac{0.1 m + 0.1 m + 0.3 m}{m + m + 3m} = \frac{3}{5} l
\end{align*}
\]

(3) \[
\vec{p}_i = \vec{p}_f \Rightarrow m_1 \vec{v}_1 + m_2 \vec{v}_2 = (m_1 + m_2) \vec{v} \\
\vec{v} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2}, \text{ same direction} \\
\vec{v} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = 0.213 \text{ m/s}
\]

(4) Before: \[ K_i = K_{total} - K_{cm} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 - \frac{1}{2} (m_1 + m_2) v^2 \]
\[ K_i = 0.00277 \approx 0.0037 \]

After: \[ K_f = 0 \]

(5) \[
\begin{align*}
\vec{p}_i &= \vec{p}_f \Rightarrow \vec{p}_i = \vec{p}_f + \vec{p}_2 \\
\vec{p}_i &= \vec{p}_1 + \vec{p}_2 \\
\vec{p}_1 &= m_1 \vec{v}_1, \vec{p}_2 &= m_2 \vec{v}_2 \\
o_x: \quad &m \vec{v} = m_1 \vec{v}_1 \cos 30^\circ + m_2 \vec{v}_2 \cos 30^\circ \\
o_y: \quad \vec{v} = m_1 \vec{v}_1 \sin 30^\circ - m_2 \vec{v}_2 \sin\theta \\
Only \ for: \ &\vec{v}_1 = \vec{v}_2 \ and \ \theta = 30^\circ: \\
\vec{v}_2 &= \frac{2 \vec{v}}{\sqrt{3}} = 9.24 \text{ m/s}
\end{align*}
\]
Exam 4

Name:___________________________________________

Student ID:_______________________________________

Discussion TA: Billeter  Blake  El-Gendy  Herring  Young

SHOW ALL WORK!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

[30 pts.] A satellite orbits the moon 1.00 \times 10^5 \text{ m} above the surface. For half of this orbit the satellite is out of communication with the earth. How long is this time?

I: \[ T = \sqrt{\frac{4\pi^2 R^3}{GM}} \]  

\[ +10 \]  

II: \[ T = \sqrt{\frac{4\pi^2 (R_{\text{moon}} + h)^3}{GM_m}} = \frac{4\pi^2 (1.74 \times 10^6 + 1.00 \times 10^5)^3}{6.67 \times 10^{-11} \times 7.35 \times 10^{22}} = 7.08 \times 10^3 \text{ s} \]  

\[ +18 \]  

III: \[ t = \frac{T}{2} = 3.54 \times 10^3 \text{ s} \text{ or } 59.0 \text{ min} \]  

\[ +2 \]  

in step I: Something wrong:  
1. in coefficient, get 5 points only;  
2. in R or M term, get zero.

in step II:  
1. use \( R_m \) or \( h \) or other as \( R \), lose 9 points;  
2. use \( M_{\text{earth}} \) or other as \( M \), lose 9 points;  
3. something wrong in calculation, lose 2 points.
SHOW ALL WORK!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

[30 pts.] A satellite orbits the moon $8.30 \times 10^4$ m above the surface. For half of this orbit the satellite is out of communication with the earth. How long is this time?

I:
$$ T = \sqrt{\frac{4\pi^2 R^3}{GM}} \quad \text{(+10)} $$

II:
$$ T = \sqrt{\frac{4\pi^2 (R_{\text{moon}} h)^3}{GM_m}} = \sqrt{\frac{4\pi^2 (1.74 \times 10^6 + 8.30 \times 10^4)^3}{6.67 \times 10^{-11} \times 7.35 \times 10^{22}}} = 6.98 \times 10^3 \text{ s} \quad \text{(+18)} $$

III:
$$ t = \frac{T}{2} = 3.49 \times 10^3 \text{ s or } 58.2 \text{ min} \quad \text{(+2)} $$

In step I: something wrong in: ① coefficient, lose 5 points; ② $R$ or $M$ term, get zero.

In step II: ① use $R_m$ or $h$ or other as $R$, lose 9 points; ② use $M_{\text{earth}}$ or other as $M$, lose 9 points; ③ something wrong in calculation, lose 2 points.
SHOW ALL WORK!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

[35 pts.] Calculate the center of mass of the object in the figure. Assume the object is a uniform sheet of thickness t and density \( \rho \).
Give both x and y coordinates for the center of mass.

1. From symmetry of the object, we derive \( CM_x = 0 \)
\( CM_x \) means Center of Mass in x direction.
\[
\frac{dm}{dV} = \rho t (B - y) \Rightarrow M_{total} = \int_{-B}^{B} \rho t (B - x^2) \, dx = \frac{4}{3} \rho t B^\frac{3}{2}
\]
\[
CM_x = \frac{1}{M_{total}} \int_{-B}^{B} x \, dm = \frac{1}{M_{total}} \int_{-B}^{B} x \cdot \rho t (B - x^2) \, dx
\]
\[
CM_x = \frac{1}{M_{total}} \left[ \int_{-B}^{B} \rho t x \, dx - \int_{-B}^{B} \rho t x^3 \, dx \right]
\]
\[
CM_x = \frac{1}{M_{total}} \left[ \frac{\rho t B^\frac{3}{2} x^2}{B} - \frac{\rho t}{4} x^4 \right]_{-B}^{B}
\]
\[
CM_x = \frac{1}{M_{total}} \left\{ \rho t B^\frac{3}{2} \left( B^2 - B^2 \right) - \frac{\rho t}{4} \left( B^4 - (-B)^4 \right) \right\}
\]
\[
CM_x = 0
\]

2. \( CM_y = \frac{1}{M_{total}} \int_{-B}^{B} y \, dm \)
\[
\frac{dm}{dV} = \rho t 2x \, dy
\]
\[
y = x^2
\]
\[
CM_y = \frac{1}{M_{total}} \int_{0}^{B} y \rho t 2y^\frac{3}{2} \, dy
\]
\[
CM_y = \frac{1}{M_{total}} \int_{0}^{B} 2 \rho t y^\frac{5}{2} \, dy
\]
\[
CM_y = \frac{1}{M_{total}} \cdot \frac{4}{5} \rho t B^\frac{5}{2}
\]
\[
CM_y = \frac{4}{3} \rho t B^\frac{5}{2}
\]
\[
CM_y = \frac{2}{5} B
\]
Exam 4

Name: ____________________________

Student ID: _______________________

Discussion TA: Billeter  Blake  El-Gendy  Herring  Young

SHOW ALL WORK!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

[35 pts.] Calculate the center of mass of the object in the figure. Assume the object is a uniform sheet of thickness t and density ρ. Give both x and y coordinates for the center of mass.

1) From symmetry of the object, we derive CMx = 0
CMx means Center of Mass in x direction.

\[
dm = \rho t (B-y) \, dx \\
\Rightarrow \sum dm = \sum_{x} \rho t (B-x^4) \, dx = \frac{8}{5} \rho t B^\frac{5}{4} \, y=x^4
\]

\[
CMx = \frac{1}{M_{\text{total}}} \sum_{x} x \, dm = \frac{1}{M_{\text{total}}} \sum_{x} \frac{\sqrt{B}}{B^\frac{5}{4}} \, \rho t (B-y) \, dx = \frac{1}{M_{\text{total}}} \sum_{x} \frac{\sqrt{B}}{B^\frac{5}{4}} \, \rho t x (B-x^4) \, dx
\]

\[
\Rightarrow \, CMx = \frac{1}{M_{\text{total}}} \left[ \int_{B^\frac{1}{4}}^{B^\frac{3}{4}} \rho t B x \, dx - \int_{-B^\frac{1}{4}}^{-B^\frac{3}{4}} \rho t B x^5 \, dx \right]
\]

\[
= \frac{1}{M_{\text{total}}} \left[ \rho t B \left( x^2 \right) \bigg|_{B^\frac{1}{4}}^{B^\frac{3}{4}} - \rho t B \left( x^6 \right) \bigg|_{-B^\frac{1}{4}}^{-B^\frac{3}{4}} \right]
\]

\[
= \frac{1}{M_{\text{total}}} \left[ \frac{1}{2} \rho t B (B^\frac{1}{2})^2 - (-B^\frac{1}{2})^2 \right] - \frac{1}{6} \rho t \left[ (B^\frac{1}{2})^6 - (-B^\frac{1}{2})^6 \right]
\]

\[
= \frac{1}{M_{\text{total}}} \left[ \frac{1}{2} \rho t B \left( B^\frac{1}{2} - B^\frac{1}{2} \right) - \frac{1}{6} \rho t \left( B^\frac{3}{2} - B^\frac{3}{2} \right) \right]
\]

\[
= 0
\]

2) \[ CM_y = \frac{1}{M_{\text{total}}} \sum_{y} y \, dm \]

\[
dm = \rho t 2xy \, dy \\
\Rightarrow \, \sum dm = \sum_{y} \rho t 2xy \, dy
\]

\[
CM_y = \frac{1}{M_{\text{total}}} \int_{0}^{B} y \, \rho t 2x \, dy
\]

\[
= \frac{1}{M_{\text{total}}} \int_{0}^{B} 2pt \, \frac{4}{9} y^\frac{5}{2} \, dy
\]

\[
= \frac{1}{M_{\text{total}}} \left[ \frac{8}{9} \rho t B^\frac{5}{2} \right]_0^B
\]

\[
= \frac{8}{9} \rho t B^\frac{5}{2}
\]

\[
= \frac{8}{9} \rho t B^\frac{5}{2}
\]

\[
= \frac{5}{9} B
\]
Problem 3:
Grading Scale:

① If you write the formula about center of mass, you get 5 points.
   For example, \( CM_x = \frac{1}{M} \int x dm \) and \( CM_y = \frac{1}{M} \int y dm \) or similar formula.

② For \( x \) coordinates of the center of mass.
   ④ Write \( dm = \rho (18-y)dx \) and get the right total mass, plus 5 points.
   ⑤ Write the right integral limits, such as \( \int_{18}^{18} dx \) and get the final result, plus 5 points.

③ For \( y \) coordinates of the center of mass.
   ⑥ Write \( dm = \rho 2xy dy \) and get the right total mass. Plus 5 points
   ⑦ Write the right integral limits. Such as \( \int_0^B dy \). Plus 5 points
   ⑧ After you have a right calculation and get the final answer, plus 10 points.
An open organ pipe is open at both ends. By experiment, resonances are found at 655 Hz, 1048 Hz, 1703 Hz and 2227 Hz as well as others. Take the speed of sound as 330 m/s.

(a) What is the largest value of the fundamental frequency allowed by the data?

(b) If there is a pressure node at each open end of the pipe, how many pressure nodes are there between the ends for the frequency 1703 Hz?

(c) How long is the pipe?

**[Solution]:**

a) \( L = \frac{n}{2} \lambda \); \( \lambda_n = \frac{2L}{n} \); \( f_n = \frac{V}{\lambda_n} = n \cdot \frac{V}{2L} = n f_1 \propto n \)  

Note (By reduction):

\[
\begin{align*}
\frac{1048}{655} &= \frac{8}{5}, & \frac{1703}{655} &= \frac{13}{5}, & \frac{2227}{655} &= \frac{17}{5}.
\end{align*}
\]

Let: \( f_1 = \frac{655}{5} = \frac{1048}{8} = \frac{1703}{13} = \frac{2227}{17} = 131 \text{ (Hz)} \)

\[
\begin{align*}
655 &= 5f_1 = f_5 \\
1048 &= 8f_1 = f_8 \\
1703 &= 13f_1 = f_{13} \\
2227 &= 17f_1 = f_{17}
\end{align*}
\]

\[
\begin{bmatrix}
655 \\
1048 \\
1703 \\
2227
\end{bmatrix}
\begin{bmatrix}
f_1 \\
f_8 \\
f_{13} \\
f_{17}
\end{bmatrix}
\]

Or By SUBTRACTION:

\[
\begin{bmatrix}
655 & > & 393 & > & 262 & > & 131
\end{bmatrix}
\]

And This is the largest one; otherwise we can not integer n's for all f's. \( f_1 = 131 \text{ (Hz)} \)

b) Now \( n = 13 \); \( \therefore \) Nodes between the ends = \( n - 1 = 12 \)

c) \( f_1 = \frac{V}{L} \)

\( L = \frac{\lambda_{f_1}}{2} = \frac{330}{2 \times 131} \approx 1.26 \text{ (m)} \)
A violin type string is clamped between supports 27.0 cm apart. The string between the supports has a mass of 0.0150 kg, and the fundamental frequency is tuned to be 440 Hz.

(a) What is the tension needed in the string? (5)
(b) What is the wavelength and frequency of the mode with 5 nodes between the clamps? (10)
(c) Using the same string and the same tension, the positions of the clamps are changed. The frequency of the new first overtone (one node between the clamps) is found to be 1012 Hz. How far apart are the clamps? (10)

(a) \( \nu = \sqrt{\frac{T}{\mu}} \Rightarrow \mu = \frac{m}{L} = 0.056 \text{ kg/m} \quad \text{2} \)

(b) \( f = \frac{3\lambda}{L} \Rightarrow \lambda = \frac{L}{3} = 0.09 \text{ m} \quad \text{2} \)
\[ f = \frac{\nu}{\lambda} \Rightarrow f = 2.64 \times 10^3 \text{ Hz} \quad \text{2} \]

(c) \( \lambda = L' \), \( \lambda = \frac{\nu}{f'} = \frac{377.6}{1012} \approx 0.37 \text{ m} \quad \text{2} \)
\[ L' = 23.6 \text{ cm} \quad \text{2} \]
An organ pipe is open at one end and closed at the other. By experiment, resonances are found at 635 Hz, 889 Hz, 1143 Hz and 1651 Hz, as well as others. Take the speed of sound as 330 m/s.

(a) What is the largest value of the fundamental frequency allowed by the data? [None of the above is the fundamental. The fundamental is lower than 635 Hz.]

(b) What is the length of the pipe?

For a general standing wave
\[ L = (2n+1) \frac{\lambda_0}{4} \quad (n = 0, 1, 2, \ldots) \]
\[ f_n = \frac{V}{\lambda_n} = \frac{V}{4L} (2n+1) \]
\[ \frac{f_{n+1} - f_n}{\Delta f} = \frac{V}{4L} [(2n+3)-(2n+1)] = \frac{V}{2L} \]
\[ \frac{L}{2\Delta f} \]

Notice that this does not depend on what we use for \( n \).

For the fundamental,
\[ L = \frac{\lambda_0}{4} = \frac{V}{2\Delta f} \]

But \( \lambda_0 = \frac{V}{f_0} \), so
\[ \frac{V}{4f_0} = \frac{V}{2\Delta f} \quad \frac{f_0}{\Delta f} = \frac{1}{2} \]

To get the largest value of \( f_0 \), we must use the largest value of \( \Delta f \) allowed by the data given.
The difference between any two frequencies must be an integer multiple of $\Delta f$. Thus, the largest value consistent with this data is $\Delta f = 254$ Hz. Notice that $508/2 = 2\Delta f$.

(Note that any value of $\Delta f$ of the form $\Delta f = \frac{1}{m}254$ Hz where $m = 1, 2, 3, \ldots$ would satisfy the requirement that each of the frequency differences above be an integer multiple of $\Delta f$. However, we are interested in only the largest allowed value of $\Delta f$.)

(a) $f_o = \frac{1}{2} \Delta f = 127$ Hz

(b) $L = \frac{v}{2\Delta f} = 0.650$ m (using $\Delta f = 254$ Hz).

Common mistakes:
- Assuming (or guessing or whatever) a value for $f_o$ without showing that it works for all the data.
- Not showing that the value obtained for $f_o$ is the largest allowed by the data.
A string is clamped between two supports 1.75 m apart. The total mass of the string free to oscillate between the supports is 0.0625 kg. The string is driven by a fixed frequency oscillator at 120 Hz, as shown in class.

(a) Calculate the values of the tension needed to produce the first four harmonics (the fundamental and the first three overtones) of the driving frequency.

(b) A new string is clamped in the same apparatus. It is found by experiment that the tension to produce the fundamental is 82.0% of that in (a). Find the total mass of the new string.

\[ \nu = \lambda f = \left( \frac{T}{\mu \lambda} \right)^{\frac{3}{2}} \]
\[ T = \mu \lambda^2 f^2 \]
\[ \mu = \frac{m}{l} \]
\[ \lambda = n \left( \frac{\lambda}{2} \right) \quad n = 1, 2, 3, \ldots \]
\[ \lambda = \left( \frac{2l}{n} \right) \]
\[ T_n = \left( \frac{m}{l} \right) \left( \frac{2l}{n} \right)^2 f^2 \]
\[ T_n = \left( 4m \frac{l^2 f^2}{n^2} \right) \]
\[ T_n = \frac{6300N}{n^2} \]

\[ \begin{align*}
T_1 & = 6300N \\
T_2 & = 1580N \\
T_3 & = 700N \\
T_4 & = 394N
\end{align*} \]

(b) \[ T \propto m \quad \Rightarrow \quad \frac{T}{T_o} = \frac{m}{m_o} \]

\[ m = m_o \left( \frac{T}{T_o} \right) = m_o (0.82) \]

\[ m = 5.13 \times 10^{-2} \text{ kg} \]
Given a wire 3.5 m long, whose mass is 0.150 kg. This wire is stretched between supports 2.50 m apart. Tension is supplied until the second harmonic standing wave occurs (2 nodes between the supports).

(a) Find the tension necessary to produce this result if the frequency of excitation is 125 Hz.

(b) Find the frequency of the third harmonic if all the conditions remain the same.
While singing in the shower, we notice that the system is resonant at certain frequencies. Consider only end walls that are 8.00 ft apart (i.e., ignore effects due to side walls, ceiling and floor).

(a) Calculate the first four frequencies at which resonant standing waves would occur between these walls. Assume the air is at 20.0°C. \( V_{sound} = 1128.6 \text{ ft/s} \)

(b) If turning on the shower and increasing the humidity lowers the density of the air by 2.50%, leaving all other quantities unchanged, find the new fundamental resonant frequency. (A numerical value for the density of air is not needed.)

\[
V = \frac{1128.6 \text{ ft/s}}{1.025} \text{ allowable wavelengths are given by the relationship:} \]

\[
\lambda = \frac{2L}{n} \quad \text{length (8 ft) and } n+1 = \text{ node!} \\
\text{number of nodes and } n = 1, 2, 3, \ldots
\]

Also, \( f = \frac{V}{\lambda} \).

\[n = 1: \lambda_1 = \frac{2 \times 8}{1} = 16 \text{ ft} \]

\[n = 2: \lambda_2 = \frac{2 \times 8}{2} = 8 \text{ ft} \]

\[n = 3: \lambda_3 = \frac{2 \times 8}{3} = \frac{16}{3} \text{ ft} \]

\[n = 4: \lambda_4 = \frac{2 \times 8}{4} = 4 \text{ ft} \]

\[
f_0 = \frac{V}{\lambda_0} = \frac{1128.6}{16} = 70.5 \text{ s}^{-1}
\]

\[f_1 = \frac{1128.6}{8} = 141.1 \text{ s}^{-1} \]

\[f_2 = \frac{1128.6}{16/3} = 212 \text{ s}^{-1} \]

\[f_3 = \frac{1128.6}{4} = 282 \text{ s}^{-1} \]

(b) \( V = \sqrt{\frac{\gamma P}{\rho}} \), so \( V \propto \sqrt{\frac{1}{\rho}} \)

\[V_0 = \sqrt{\frac{P_0}{\rho_0}} \quad V_1 = \sqrt{\frac{P_1}{\rho_1}} \quad V_2 = \sqrt{\frac{P_2}{\rho_2}} \]

now: \( P_f = P_0 (1 - 0.025) \)

\[P_f = 0.975 P_0 \quad V_f = V_0 \sqrt{\frac{P_2}{\rho_2}} = V_0 \frac{1}{\sqrt{0.975}} \]

\[V_0 = 1128.6 \text{ ft/s} \quad \text{So, } V_f = \frac{1128.6}{\sqrt{0.975}} = 1143 \text{ ft/s} \]

From part (a): \( f_0 = \frac{V}{\lambda_0} = \frac{1143}{16} = 71.4 \text{ s}^{-1} \)
4A. A tuning fork placed over an open vertical tube partly filled with water causes strong resonances when the water surface is 8 cm and 28 cm from the top of the tube and for no other positions. The speed of sound in the air in the room is 330 meters/sec. What is the frequency of the tuning fork?

For a closed tube, we have \( n \approx \frac{2n+1}{2} \), but this is only an approximation; the antinode is not right at the open end of the tube, i.e.

\[
\begin{align*}
\text{open} & : \quad \lambda = 2L \\
\text{closed} & : \quad \lambda = \frac{2L}{n}
\end{align*}
\]

Then \( \lambda + x = \frac{2L}{n} \), \( \lambda - x = \frac{2L}{n} \)

\( \Rightarrow \) \( \lambda + x - \lambda + x = \frac{2L}{n} - \frac{2L}{n} \) \( \Rightarrow 28cm - 8cm = 20cm = \frac{2L}{n} \)

\( \lambda = 14m \)

Then this can be gotten diagrammatically as follows:

\[
\begin{align*}
\lambda & = 20cm, \quad \lambda = 40cm \Rightarrow 14m \\
\text{then} \quad f & = \frac{v}{\lambda} = \frac{330 \text{ m/s}}{14 \text{ m}} \\
& = 23.57 \text{ Hz}
\end{align*}
\]
An organ pipe is open at one end, and closed at the other. It is adjusted so that the first overtone (the first resonant frequency higher than the fundamental) occurs when the tube is 2.25 m long. Take the velocity of sound to be 330 m/s.

(a) Find the wavelength of the first overtone.
(b) Find the frequency of the first overtone.
(c) Find the frequency of the fundamental.
(d) If the velocity of sound is increased by 1.00%, find the new frequency of the first overtone.

Report numerical answer to three significant figures.

The modes of an organ pipe which is closed at one end and open at the other are shown on page 326 of your text. For this problem, you need to know the wavelength of the fundamental and of the first overtone. The quantity \( l \) is given as 2.25 m, and \( v = \frac{330 m}{s} \).

- Fundamental
- First overtone

Remember that a full wavelength looks like

\[ \lambda = \frac{3}{4} \lambda \]

\[ \lambda = \frac{3}{4} \lambda \]
(a) Wavelength of first overtone:
\[ l = \frac{3}{4} \lambda \implies \lambda = \frac{4}{3} l = \frac{4}{3} (2.25\text{ m}) = 3.00\text{ m} \]

(b) Frequency of first overtone:
\[ v = \lambda \nu \implies \nu = \frac{v}{\lambda} = \frac{330\text{ m/s}}{3.00\text{ m}} = 110\text{ Hz} \]

(c) Frequency of fundamental:
First find the wavelength of the fundamental.
\[ l = \frac{2}{4} \implies \lambda = 4l = 4 (2.25\text{ m}) = 9.00\text{ m} \implies v = \frac{v}{\lambda} = \frac{330\text{ m/s}}{9.00\text{ m}} \]
\[ = 36.7\text{ Hz} \]

(d) The new velocity of sound in air is
\[ (1.01)(330\text{ m/s}) = 333\text{ m/s} \implies \nu = \frac{v}{\lambda} = \frac{333\text{ m/s}}{3.00\text{ m}} = 111\text{ Hz} \]

Remember that the pipe length is fixed. Specifying fundamental, 1 overtone, etc. tells you how many wavelengths "fit" in pipe. The open end is always an antinode; the closed end is always a node.
A string 1.34 m long and clamped at both ends, is excited at 250 Hz. The mass density of the string is 0.15 kg/m. Three nodes appear between the supports. 

(a) Calculate the fundamental frequency. (b) Calculate the tension in the string.

(c) Now the tension is increased, until there are only two nodes between the supports for the same frequency of excitation. Find the new fundamental frequency, and the new tension.

In general, \( y = A \sin(kx + \omega t + \phi) \), where \( k = \frac{2\pi}{\lambda} \). Due to boundary conditions (clamped at both ends) we know that:

\[
\begin{align*}
Kx \pm a & \rightarrow \frac{2\pi}{\lambda} x = n\pi \rightarrow \lambda_n = \frac{2\pi}{n} \quad n = 1, 2, 3, ... \\
\phi_n & = \frac{v}{2\lambda} \\
\phi_1 & = \frac{v}{2\lambda} (\text{fundamental}) \quad \lambda_3 \\
\phi_4 & = \frac{4v}{6\lambda} (3 \text{ nodes}) \quad \phi_1 = \frac{4\phi_1}{4} \rightarrow \phi_1 = \frac{1}{4} \phi_4
\end{align*}
\]

a) \( f_1 = \frac{250}{2\lambda_1} \quad \text{Hz} \quad \lambda_2 = \frac{62.5}{f_1} \rightarrow 10 \text{ POINTS} \)

b) \( T = \frac{\mu V^2}{2} \) where \( V = \frac{2\lambda f_2}{\lambda} f_2 = \frac{2\lambda f_3}{\lambda} f_3 = \frac{1}{2} \lambda f_4 = \cdots \)

\[
T = (0.5) (2(1.34)(62.5)) \quad \text{N} = \frac{4811N = T}{5 \text{ POINTS}}
\]

c) \( f_3' = \frac{3V'}{2\lambda}, \quad f_3' = \frac{v'}{2\lambda} \rightarrow f_4' = \frac{1}{3} f_3' = \frac{\text{N} = 88.5 \text{ Hz}}{5 \text{ POINTS} \text{ FOR } f_1' \& T' \text{ EACH}}
\]

Common Errors: (1) Not knowing what nodes are. (2) Mixing frequencies and wavelengths in calculation of \( V \), (e.g., \( V = f_1 f_2 ? \)) (3) Using \( V = 300 \text{ m/s} \) and \( v = 300 \text{ m/s} \). This problem was generally done well. 121 out of 116 earned 25 pts.
A violin string 0.700 m long is clamped at both ends. The mass of the string is 12.0 gram.

(a) Find the tension necessary so that the fundamental mode will be at a frequency of 440 Hz.
(b) Set the origin of your coordinate system \((x = 0)\) at the midpoint of the string, as shown. Write, as completely as possible, the function describing the waves on this string for the fundamental and the first two overtones. [Hint: Draw pictures of the waves for these three situations.]

\[
\begin{align*}
(1) \quad & \nu = \lambda f = \left( \frac{T}{\mu} \right)^{1/2} = \left( \frac{Tl}{m} \right)^{1/2} \\
(2) \quad & l = \pi \left( \frac{1}{2}, 1, 2, \ldots \right) \\
(3) \quad & T = 4 \left( 0.700 \text{ m} \right) \left( 440 \text{ Hz} \right)^2 \left( 1.2 \text{ kg} \right) \\
\end{align*}
\]

\[
T = 6.50 \times 10^2 \text{ N}
\]

\[
\begin{align*}
(1) \quad & \nu = \omega_1 \frac{k_1}{k_2} = \omega_2 \frac{k_2}{k_3} = \lambda f \\
(2) \quad & k_n = \frac{2 \pi}{\lambda_n} = \frac{2 \pi}{n \lambda} \quad n = 1, 2, 3, \ldots \\
(3) \quad & \omega_n = 2 \pi f_n = \omega_1 = 2 \pi \left( 440 \text{ Hz} \right) \\
& k_1 = \frac{2 \pi}{1.2 \text{ m}} \quad k_2 = \frac{4 \pi}{0.4 \text{ m}} \quad k_3 = \frac{6 \pi}{0.5 \text{ m}}
\end{align*}
\]

\[
\begin{align*}
& y_1(x, t) = \alpha_1 \cos \left( k_1 x \right) \cos \left( \omega_1 t \right) \\
& y_2(x, t) = \alpha_2 \sin \left( k_2 x \right) \cos \left( \omega_2 t \right) \\
& y_3(x, t) = \alpha_3 \cos \left( k_3 x \right) \cos \left( \omega_3 t \right)
\end{align*}
\]

\[
\begin{align*}
& y_1(x, t) = \alpha_1 \cos \left( \frac{2 \pi x}{1.2 \text{ m}} \right) \cos \left( 2 \pi t, 440 \text{ Hz} \right) \\
& y_2(x, t) = \alpha_2 \sin \left( \frac{4 \pi x}{0.4 \text{ m}} \right) \cos \left( 4 \pi t, 440 \text{ Hz} \right) \\
& y_3(x, t) = \alpha_3 \cos \left( \frac{6 \pi x}{0.5 \text{ m}} \right) \cos \left( 6 \pi t, 440 \text{ Hz} \right)
\end{align*}
\]

\[
3.5
\]
4 A. A violin string is given a tension of 200 N. The string has a mass density of 0.004 kg/m.
(a) Find the velocity of waves in this string.
(b) Find the wavelength of a 440 Hz wave in this string.
(c) If the string is attached between supports 0.5 m apart, find a general formula for the allowed wavelengths in the system.
(d) For the three longest wavelengths you obtain in (c), find the frequency, in Hz.

\[ V = \sqrt{\frac{200 \text{ N}}{0.004 \text{ kg/m}}} = \sqrt{\frac{2 \times 10^4}{4 \times 10^{-3}}} = \sqrt{5 \times 10^5} = \sqrt{5} \times 10^2 \text{ m/s} \\
= 2.24 \times 10^2 \text{ m/s} \]

b. \[ V = \lambda \nu \quad \lambda = \frac{5 \times 10^2}{4 \times \lambda_{10^2}} = \frac{5}{4} = 1.25 \text{ m} \]

Since the ends of string at the ends are fixed, the allowed or natural standing waves must have nodes at these endpoints. The longest wavelength that meets this condition is:

\[ \lambda = \frac{2l}{n} \quad \text{wavelength} \]
\[ l = \text{lengths between fixed endpoints} \]

Other wavelengths are then:

\[ \lambda = \frac{2l}{n} \quad (n=1, 2, 3, \ldots) \]

d. 3 longest are:

\[ \lambda_1 = \frac{1m}{n=1} \]
\[ \lambda_2 = \frac{1.5m}{n=2} \]
\[ \lambda_3 = \frac{0.333m}{n=3} \]

From \[ \lambda V = V = 224 \text{ m/s} \]
\[ V_1 = 224 \text{ Hz} \]
\[ V_2 = 448 \text{ Hz} \]
\[ V_3 = 672 \text{ Hz} \]
SECOND MIDTERM

Name (print): ___________________________  Name (signed): ________________

Discussion Instructor (circle one): Chen  Emerson  Iguchi  Stoops

Discussion Section #: ________________

SHOW ALL WORK!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES.
Use the conversion constants and data given on the front page.

A violin string is tuned so that the fourth harmonic (three nodes between the supports) has a frequency of 1900 Hz. The tension in the string is 900 N. The supports are 0.30 m apart.

(a) Calculate the mass density of the string.

(b) If the tension is increased by 1.00%, calculate the frequency of the fundamental.

(c) Write a complete expression for the displacement $y$, as a function of time and position, for the fourth harmonic described above. Choose $x = 0$ at the mid-point of the string.

10 pts. a.)  \[ f_n = \frac{n}{2L} \sqrt{\frac{F}{\mu}} \]

\[ f_4 = \frac{4}{2L} \sqrt{\frac{F}{\mu}} = \quad \Rightarrow \mu = \left(\frac{4}{2L} \frac{F}{f_4}\right)^2 = 0.0111 \text{ kg/m} \]

5 pts. b.)  \[ F' = F + 0.01F = 1.01F \]

\[ f_1 = \frac{1}{2L} \sqrt{\frac{1.01 \times 922}{0.0111}} = 477 \text{ Hz} \]

10 pts. c.)  \[ y = 2A \sin kx \cos \omega t \]

Where  \[ k = \frac{4\pi}{L}, \quad \omega = 2\pi f_4 = 3900 \pi \]
A violin string is clamped between supports 0.45 m apart.

**Problem 1(a)** If the string is tuned to a fundamental of 440 Hz with a tension of \(2.75 \times 10^3\) N, calculate the mass density of the string.

**Problem 1(b)** If the tension is increased to \(2.85 \times 10^3\) N, calculate the new fundamental frequency.

**Problem 1(c)** If it is desired to have the mode with two nodes between the supports be at a frequency of 1285 Hz, what tension is needed?

\[
\lambda = \frac{1}{4} \text{m}, \quad f = \frac{1}{440\text{Hz}}
\]

\[
v = \sqrt{\frac{T}{\mu}}
\]

\[
v = (0.90 \text{ m})(440\text{Hz}) = 386 \text{ m/s}
\]

\[
\mu = \frac{T}{v^2} = \frac{2.75 \times 10^3 \text{N}}{(386 \text{ m/s})^2} = 17.54 \times 10^{-3} \text{kg/m}
\]

\[
f = 440\text{Hz} \left( \frac{2.85 \times 10^3 \text{N}}{3.75 \times 10^3 \text{N}} \right)^{1/2} = 448 \text{Hz}
\]

\[
L = \frac{3}{4} \lambda, \quad \lambda = \frac{2}{3} L
\]

\[
v = \lambda f = \left(\frac{2}{3} L\right)(1285\text{Hz}) = 320.5 \text{ m/s}
\]

\[
v = \sqrt{\frac{T}{\mu}}, \quad \mu = 12.54 \times 10^{-3} \text{kg/m}
\]

\[
T = 2.61 \times 10^3 \text{N}
\]
A tube is 1.45 m long and open at both ends. Resonances are found at 234 Hz, 585 Hz, 936 Hz, and 1404 Hz among others.

(a) Find the largest value of the fundamental allowed by these data.
(b) How many nodes for pressure are there for 1404 Hz—not counting the ends.
(c) Calculate the velocity of sound for this system.

\[ a) \quad \frac{234}{351} > 1, \quad \frac{585}{351} > 1, \quad \frac{936}{468} > 1.17 \text{ Hz} \]

\[ a) \quad \text{The Fundamental Frequency} = 117 \text{ Hz.} \]

(It is the largest; notice that 117, etc divide the other frequencies, but 117 is the one which satisfies the problem).

\[ b) \quad n = \frac{1404}{117} = 12 \]

\[ \Rightarrow \text{number of nodes} = 11 \]

\[ c) \quad v = \lambda f \quad , \quad \lambda = 2L \quad , \quad f \text{ is the fundamental} \]

\[ = f, (2L) \]

\[ = (117 \cdot \frac{1}{2})(2(1.45 \text{ m})) \]

\[ = 339 \text{ m/s} \]
SECOND MIDTERM

Name (print) \[ \text{Jim} \] Name (signed) \\

Discussion Instructor (circle): Condella Guilkey Leong Nott Paul Zhang

Discussion Section # \[ 1, 9 \]  

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

A traveling wave is described by the function

\[
y = (1.73 \text{ mm}) \sin \left( 27.0x + 5720t + \frac{\pi}{6} \right)
\]

Except where shown, all distances are in meters. Other quantities are in the usual and appropriate units.

(a) Calculate the magnitude of the velocity of the wave.

\[
V = \frac{\lambda}{T} = \frac{\lambda}{27 \text{ m}} = \frac{5720 \text{ s}^{-1}}{27 \text{ m}^{-1}} = 212 \text{ m/s}
\]

(b) Specify in words, the direction of the wave.

In the negative x direction, or to the left

(c) Calculate the wavelength.

\[
\lambda = \frac{2\pi}{k} = \frac{2\pi}{27 \text{ m}^{-1}} = 2.33 \text{ m}
\]

(d) Calculate the frequency in Hertz.

\[
f = \frac{v}{\lambda} = \frac{5720 \text{ s}^{-1}}{27 \text{ m}^{-1}} = 212 \text{ Hz}
\]

(e) At \( t = 0 \), calculate the first positive value of \( x \) for which \( y = 0 \).

\[
y = 0 \quad \text{when} \quad (27x + \frac{\pi}{6}) = n\pi
\]

\[
x = \frac{n\pi - \frac{\pi}{6}}{27 \text{ m}^{-1}}
\]

Taking \( n = 1 \)

\[
x = \frac{5\pi}{6(27 \text{ m}^{-1})} = 0.0970 \text{ m}
\]
SECOND MIDTERM

Name (print) ZHANG TIAN Name (signed) ________________________________

Discussion Instructor (circle): Condella Guiley Leong Nott Paul Zhang

Discussion Section # ______

SHOW ALL WORK!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

An open tube is arranged with one end in a beaker of water. When a tuning fork (1024 Hz) is held nearby, resonances are observed for d = 3.00 in, 9.45 in, 15.90 in and 22.35 in. Take the speed of sound as 1100 ft/s.

(a) For d = 15.90 in, calculate the frequency of the fundamental. Include the "end correction" calculated from the information given.

(b) If helium is added to the tube so that its density is reduced from 1.29 g/l to 1.17 g/l, calculate the new value of the fundamental in (a). Consider only the change in density of the gas in the tube.

\[ f = \frac{(2n+1)^2}{4L_{n+1}} = \frac{(2n+3)^2}{4L_{2n+3}} \]

Let \( L_{2n+3} - L_{2n+1} = \left[(2n+3) - (2n+1)\right] \frac{v_f}{4} = 2d_1 \)

3.00 > 6.45 \[ d_1 = \frac{1}{2} \times 6.45 = 3.225 \implies x = 3.00 - 3.225 = 0.225 \text{ in} \]

9.45 > 6.45 \[ +3 \text{ points} \]

15.90 > 6.45 \[ +2 \text{ points} \]

22.35 > 6.45 \[ +5 \text{ points} \]

\[ a) \quad f_0 = \frac{v_f}{4L_1} = \frac{5}{4 \times 16.125} = \frac{1150 \times 12}{4 \times 16.125} = 205 \text{ Hz} \]

OR for \( L_1 = 16.125 \text{ in} \) \( n=5 \) so \( f_0 = \frac{5}{5} = \frac{1024}{5} = 205 \text{ Hz} \)

\[ b) \quad \frac{V_{\text{new}}}{V_{\text{old}}} = \sqrt{\frac{\rho_{\text{old}}}{\rho_{\text{new}}}} \quad \frac{f_{\text{new}}}{f_{\text{old}}} = \frac{V_{\text{new}}}{V_{\text{old}}} = \sqrt{\frac{1.29}{1.17}} \quad f_{\text{new}} = \sqrt{\frac{1.29}{1.17}} \times 205 = 215 \text{ Hz} \]

\[ +8 \text{ points} \]

\[ +2 \text{ points} \]
A wave on a string is described by the solution
\[ y = (6.25 \times 10^{-3}) \sin(575x + 425t + 0.87) \]

Distances are in meters, time in seconds, and other quantities have the usual and appropriate SI units.

(a) Calculate the velocity of the wave (including direction). (5 pt)
(b) Calculate the maximum value of the transverse velocity. (5 pt)
(c) At \( t = 0 \), calculate the smallest positive value of \( x \) for which the displacement is \( 3.00 \times 10^{-3} \) m. (5 pt)
(d) Calculate the maximum value of the displacement. (5 pt)
(e) Calculate the frequency \( f \) (the number of peaks per second passing a given point). (5 pt)

Solution:

(a) \[ y = A \sin (kx - \omega t + \delta) = 6.25 \times 10^{-3} \sin(575x - 425t + 0.87) \text{ cm} \]

\[ \therefore \text{ it propagates to the left. (Ref. Fig.1) } \]
\[ v = \frac{\omega}{k} = \frac{425}{575} = 0.739 \text{ m/s} \]

(b) \[ v_y = \frac{dy}{dt} = 425 \times 6.25 \times 10^{-3} \cos(575x + 425t + 0.87) \]

\[ \therefore v_{y\text{max}} = 2.66 \text{ m/s} \]

(c) \[ y_{t=0} = 6.25 \times 10^{-3} \sin(575x + 0.87) = 3.00 \times 10^{-3} \]

\[ \therefore \sin(575x + 0.87) = 0.480 \]

\[ 575x + 0.87 = \pi - \sin^{-1} 0.480, \]
\[ x = 3.08 \times 10^{-3} \text{ m} \text{ (Fig.2)} \]

(d) \[ y_{\text{max}} = 6.25 \times 10^{-3} \text{ m} \]

(e) \[ \omega = 425, \quad f = \frac{\omega}{2\pi} = 67.6 \text{ (Hz)} \]
A wave on a string is described by the solution:

\[ y = \frac{(3.25 \times 10^{-3}) \cos (42.7x + 57.5t + 0.25)}{A \frac{A}{k} \frac{\omega}{\varphi}} \]

All distances are in meters, time in seconds, and all other quantities the usual and appropriate SI units.

(a) Calculate the velocity of the wave, including direction.
(b) Calculate the frequency, \( f \), in Hz.
(c) Calculate the maximum value of the transverse velocity.
(d) Calculate the wavelength.
(e) Calculate the maximum value of the displacement.

a) \( \frac{u_x}{k} = \frac{w}{k} = 1.35 \text{ m/s} \quad 4 \)

b) \( f = \frac{w}{2\pi} = 9.15 \text{ Hz} \)

c) \( v_y \text{ Max} = \omega A = 0.187 \text{ m/s} \)

d) \( \lambda = \frac{2\pi}{k} = 0.147 \text{ m} \)

e) \( A = 3.25 \times 10^{-3} \text{ m} \)
FIRST MIDTERM

Name (print) MOLINA Name (sign) ______________

Discussion Instructor (circle one): DeTienne Named Molina Paul Smith Zhang

Discussion Section # ______________

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES;
Use the conversion constants and data given on the front page.

A wave on a string is described by the function

\[ y = (4.75 \times 10^3) \sin (75.0x + 32.0t - 0.35) \]

Distances are in meters, time in seconds, and other quantities have the usual and appropriate SI units.

(a) Calculate the velocity of the wave (including direction).

\[ v = (\omega/k) = 4.27 \times 10^4 \text{ m/s} \text{ to the LEFT (negative x direction)} \]

(b) Calculate the maximum value of the transverse velocity.

\[ \left( \frac{dy}{dx} \right)_{\text{max}} = \omega A = 1.520 \times 10^5 \text{ m/s} \]

(c) Calculate the maximum value of the displacement.

\[ (y_x)_{\text{max}} = A = 4.75 \times 10^3 \text{ m} \]

(d) Calculate the frequency \( f \) (the number of peaks per second past a given point).

\[ f = \left( \frac{\omega}{2\pi} \right) = 5.09 \text{ (1/sec)} \]

(e) At \( x = 0 \), calculate the smallest positive value of time, such that the displacement is \( 2.00 \times 10^3 \text{ m} \).

\[ y_0 = A \sin (\omega t + \phi) \]

\[ \Rightarrow t = \frac{1}{\omega} \left[ \sin^{-1} (y_0/A) - \phi \pm 2\pi n \right] \quad n = 0, 1, 2, \ldots \]

\[ t_{\text{min}} = 2.45 \times 10^{-2} \text{ s} \]
FIRST MIDTERM

Name (print)  Prabasaj Paul  Name (signed)

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Discussion Section #

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

A wave on a string can be described by the solution

\[ y = (1.30 \times 10^{-3}) \sin (65.0 \times 1 + 2400t) \]

All quantities have the usual and appropriate SI units.

(a) Calculate the velocity of the wave (including direction).

\[ v = \frac{\omega}{k} = 36.9 \text{ m/s} \quad \text{in} \quad \text{ve. x dir.} \]

(b) Calculate the frequency (in Hertz).

\[ f = \frac{\omega}{2\pi} = 38.2 \text{ Hz} \]

(c) Calculate the wavelength.

\[ \lambda = \frac{2\pi}{k} = 9.67 \times 10^{-2} \text{ m} \]

(d) If the tension in the string is 45 N, calculate the mass density of the string.

\[ \mu = \frac{F}{v^2} = 3.30 \times 10^{-2} \text{ kg/m} \]

(e) Calculate the first positive value of t (t > 0) for which the displacement has its maximum negative value. At \( \lambda = 0 \).

\[ 2400 \times t = \frac{3\pi}{2} \Rightarrow t = 1.963 \times 10^{-3} \text{ s}. \]
First Midterm

The function below describes a wave traveling on a stretched string. (x is in meters and t in seconds.)

\[ y = (1.50 \text{ mm}) \sin(9.90x + 125t - \frac{\pi}{3}) \]

\[ y = A \sin \left( \nu x - \omega t + \phi \right) \]
\[ \nu = 9.9 \quad \omega = 125 \]
\[ \nu < 0 \]

(a) Calculate the wavelength of the wave.

\[ \lambda = \frac{2\pi}{k} = 6.35 \times 10^{-1} \text{ m} \]

(b) Calculate the velocity of the wave, including its direction.

\[ v = \frac{\omega}{k} = 12.6 \text{ m/s} \quad \text{negative direction} \]

(c) Calculate the transverse velocity, giving its correct units and direction, for \( x = +2.00 \text{ m} \) and \( t = +3.00 \text{ s} \).

\[ v_x = \frac{dy}{dt} = 125 \times 1.5 \times 10^{-3} \cos\left(9.90 \times 2 + 125 \times 3 - \frac{\pi}{3}\right) = -9.27 \times 10^{-2} \text{ m/s} \]

(d) Calculate the period \( T \) for the wave.

\[ T = \frac{2\pi}{\omega} = 5.03 \times 10^{-2} \text{ s} \]

(e) If the string has a linear mass density of 0.020 kg/m, calculate the tension in the string.

\[ \sigma = \sqrt{\frac{T}{\mu}} \quad T = V^2 \mu = 3.18 \text{ N} \]
A violin string with 30.0 cm between supports is tuned to a fundamental frequency of 440 Hz. The seventh harmonic is generated (six nodes between the supports not including those of the supports, i.e., 7 antinodes).

(a) Calculate the speed of the waves on the string.
(b) Find the frequency AND wavelength of the seventh harmonic.
(c) Determine the tension needed if the 30.0 cm of string has a mass of 17.0 grams.

\[ f_1 = \frac{1}{2L} \nu \]
\[ \nu = 2L f_1 = 2 \times 0.3 \times 440 = 264 \text{ m/s} \]

\[ f_n = \frac{n}{2L} \nu = n f_1 = 3080 \text{ Hz} \]
\[ \lambda = \frac{\nu}{f_1} = \frac{264}{3080} = 0.0857 \text{ m} = 8.57 \times 10^{-2} \text{ m} \]

\[ \nu = \sqrt{\frac{E}{\mu}} \Rightarrow F = \nu^2 \mu = 264^2 \times \frac{17 \times 10^{-3}}{0.3} = 3949 \text{ N} = 3.95 \times 10^3 \text{ N} \]
SECOND MIDTERM

Name (print) ______________________________ Name (signed) __________________________

Discussion Instructor (circle): Basko  Chakhbazian  DiCarlo  Romer  Wei  Zlatkov

Discussion Section # _____

SHOW ALL WORK!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

Resonances are observed for an organ pipe open at one end and closed at the other for the following frequencies (none of these is the fundamental):

291 Hz
679 Hz
1067 Hz
1649 Hz

(a) Calculate the largest value of the fundamental frequency (in Hz) permitted by this data.
(b) If the speed of sound is taken to be 330 m/s, calculate the effective length of this organ pipe.
(c) If helium is mixed with air to reduce its density by 10.0%, determine the new value of the 291 Hz resonance considering only the density change.

\( f_n = \frac{2n+1}{4L} \), the largest common denominator

\( n = \{291, 679, 1067, 1649\} \) is 97

So \( f_1 = 97.0 \text{ Hz} \)

\( f_1 = \frac{V}{4L} \), \( L = \frac{V}{f_1} = \frac{330}{4 \times 97.0} = 0.851 \text{ m} \)

\( f \propto \frac{1}{\sqrt{\rho}} \), \( f' \propto \frac{1}{\sqrt{0.9}} \)

So \( f' = \frac{f}{\sqrt{0.9}} = \frac{291}{\sqrt{0.9}} = 307 \text{ Hz} \)
FIFTH MIDTERM

SHOW ALL WORK!!!

Use the conversion constants and data given on the front page.
Not all given data may be necessary to solve the problems.

1. A wave is described by \( \mathbf{x} = A \sin \left( kx + \omega t + \phi \right) \).

(a) [3 pts.] the wavelength \( \lambda \),
(b) [3 pts.] the frequency \( \omega \),
(c) [3 pts.] the amplitude \( A \),
(d) [3 pts.] the period \( T \),
(e) [3 pts.] the angular frequency \( \omega \), and
(f) [3 pts.] the angular wave number \( k \).

a) \( k = \frac{2\pi}{\lambda} \rightarrow \lambda = \frac{2\pi}{k} \)
b) \( \omega = 2\pi f \rightarrow f = \frac{\omega}{2\pi} \)
c) \( A = 0.40 \text{ m} \)
d) \( \omega = \frac{2\pi}{T} \rightarrow T = \frac{2\pi}{\omega} \)
e) \( \omega = 2.0 \times 10^9 \)
f) \( k = 3.0 \text{ rad/m} \)

2. [5 pts.] If you take a grandfather clock to Mars, will it run faster or slower than on Earth? The diameter of Mars is 0.53 times the diameter of the Earth, and its mass is 0.11 times the mass of the Earth.

A grandfather clock is a physical pendulum with \( \omega = \sqrt{\frac{g}{L}} \) and \( T = \frac{2\pi}{\omega} \), so \( T = \frac{2\pi}{\sqrt{g}} \).

\[
\frac{g_{\text{Mars}}}{g_{\text{Earth}}} = \frac{GM_{\text{Mars}}}{GM_{\text{Earth}}} = \frac{0.11}{(0.53)^2} \frac{L_{\text{Earth}}}{R_{\text{Earth}}} = 0.39 \frac{L_{\text{Earth}}}{R_{\text{Earth}}} = 0.39 \cdot g
\]

\( g_{\text{Mars}} < g \) so \( T_{\text{Mars}} > T_{\text{Earth}} \)

The grandfather clock will run slower.
FIFTH MIDTERM

SHOW ALL WORK!!!!
Use the conversion constants and data given on the front page.
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A standing wave on a string with fixed ends has four nodes (not counting the ends). The frequency of this mode is 120 Hz.

(a) (7 pts.) Sketch the shape of the wave.
(b) (9 pts.) Find the fundamental frequency of the string.
(c) (9 pts.) If the tension in the string is reduced by a factor of 9, what is the new fundamental frequency?

\[ \lambda = \frac{2L}{5} \quad \therefore \nu = \frac{f}{\lambda} = f \cdot \frac{2L}{5} \]

\[ f_n = \frac{n}{2L} \nu \quad \text{for the fundamental frequency } n = 1 \]

\[ f_1 = \frac{1}{2L} \cdot \frac{2L}{5} \nu = \frac{1}{5} \nu \]

\[ f_1 = \frac{120 \text{ Hz}}{5} = 24 \text{ Hz} \]

\[ \nu_0 = \sqrt{\frac{T}{\mu}} \quad \therefore \nu_0 = \sqrt{\frac{24 \text{ Hz}}{3}} \quad \nu_0 = \frac{1}{3} \nu_0 \]

\[ f_{1/4} = \lambda \nu_0 = \frac{1}{8} \nu_0 = \frac{1}{8} f_1 = \frac{24 \text{ Hz}}{8} = 3 \text{ Hz} \]

\[ f_{1/4} = \frac{24 \text{ Hz}}{8} = 3 \text{ Hz} \]
SHOW ALL WORK!!!!

Use the conversion constants and data given on the front page.
Not all given data may be necessary to solve the problems.

A uniform rod with a length $L = 60$ cm and a mass $m = 3.0$ kg is pivoted about one end and is hanging vertically down at rest. The free end of the rod is given an initial speed $v = 0.30$ m/s.
Assume it undergoes simple harmonic motion in a vertical plane, and determine its

(a) $[8\text{ pts.}]$ period, $T$
(b) $[8\text{ pts.}]$ total energy (let the potential energy be zero at the lowest point), $E$
(c) $[9\text{ pts.}]$ maximum angular displacement in degrees.

(Note: the moment of inertia of a uniform rod about an axis through one end is $I = \frac{1}{3}mL^2$.)

A. $\omega = \sqrt{\frac{mgd}{I}}$ where $d = \frac{L}{2}$
   \[\text{so } T = 2\pi \frac{1}{\omega} = 2\pi \sqrt{\frac{3I}{2mg}} = 1.27\text{s}\]

B. Energy is constant, and at the bottom it is all rotational kinetic energy:
   \[E = \frac{1}{2}I\omega^2 \text{ (where } \omega \text{ is the angular speed at the bottom, not the frequency of the pendulum)}\]
   \[E = \frac{1}{6}mL^2 \left(\frac{\omega}{L}\right)^2 = 0.045J\]

C. $\theta$ will be max. when all the energy is in potential form:
   \[mgh = 0.045J, \text{ where } h \text{ is the difference in height of the center of mass.}\]
   \[\cos \beta = \frac{\frac{L}{2} - h}{\frac{L}{2}} \Rightarrow 0 = 5.79^\circ\]

\[\Theta(\theta) = \Theta_0 \sin(\omega t)\]
\[\frac{d\theta}{dt} = \omega \Theta_0 \cos(\omega t)\]
\[\text{so } \frac{d^2\theta}{dt^2} = \omega^2 \Theta_0 \cos(\omega t) \text{ but } \omega^2 = \frac{\Theta_0 m}{L} \text{ so } \omega^2 = \frac{\Theta_0 m}{L \left(\frac{3I}{2mg}\right)} = 5.79^\circ\]
SHOW ALL WORK!!!!
Use the conversion constants and data given on the front page.
Not all given data may be necessary to solve the problems.

Two stars of equal mass \( m = 2.00 \times 10^4 \text{M}_\odot \) are revolving in a circular orbit about their center of mass (see figure). Assume the distance between the stars is \( d = 10.0 \text{ AU} \). (Here \( M_\odot = 1.99 \times 10^{33} \text{ kg} \) is the mass of the Sun and 1 AU = 1.50 \times 10^{11} \text{ m} \) is one astronomical unit). Find

(a) \[ 9 \text{ pts.} \] the magnitude of the force of gravity exerted by one star on the other,
(b) \[ 8 \text{ pts.} \] the orbital speed in km/s, and
(c) \[ 8 \text{ pts.} \] the orbital period in days.

\[ \begin{align*}
(a) F &= \frac{Gm_1m_2}{d^2} = \frac{6.67 \times 10^{-11} \times (200 \times 1.99 \times 10^{30})^2}{(10.0 \times 1.50 \times 10^{11})^2} = 4.70 \times 10^{28} \text{ N} \\
&= 4.70 \times 10^{28} \text{ N} \\
(b) \quad \mathbf{v} &= \sqrt{\frac{FR}{m_1}} = \sqrt{\frac{2 \pi d}{2m_1}} = \sqrt{\frac{4.70 \times 10^{28} \times 1.0 \times 1.50 \times 10^{11}}{2 \pi \times 2\pi \times 1.99 \times 10^{30}}} \\
&= \frac{29.8 \text{ km/s}}{2} \\
&= 29.8 \text{ km/s} \\
(c) \quad T &= \frac{2 \pi R}{v} = \frac{\pi d}{v} = \frac{3.14 \times 1.0 \times 1.50 \times 10^{11}}{29.8 \times 10^3} = 1.58 \times 10^7 \text{ s} = 183 \text{ days}.
\end{align*} \]

For those who use 1AU = 150 \times 10^3 \text{ m}.
(a) \( F = 4.70 \times 10^{28} \text{ N} \)
(b) \( v = 9.42 \text{ km/s} \)
(c) \( T = 5787 \text{ days} \).