Lecture 10: Center of Mass

Physics 2210
Fall Semester 2014
Today's Concepts:

a) Finding the Center of Mass  
b) Using the Center of Mass
Unit 10: Prelecture Feedback

• Open homeworks earlier.
  • As of now, all homeworks until the end of the semester are 'open'.

• Make checkpoint answers available the same day.
  • They're available at 11 AM the same day. Flexibility for people with emergencies.

• Checkpoint question with men pulling pole
• Homework-type questions
• Calculus practice, especially multivariable integration
Center of Mass

(for Discrete Distributions)

\[ \overrightarrow{R}_{CM} = \frac{1}{M_{Total}} \sum_{i=1}^{N} m_i \overrightarrow{r}_i \]

(for Continuous Distributions)

\[ \overrightarrow{R}_{CM} = \frac{1}{M_{Total}} \int \overrightarrow{r} \, dm \]

(for System of Solid Objects)

\[ \overrightarrow{R}_{CM} = \frac{1}{M_{Total}} \sum_{i=1}^{N} M_i \overrightarrow{R}_{CM,i} \]
Example

A system consists of three particles located as shown in the Figure. Find the center of mass of the system. \( m_1 = m_2 = 1 \text{ kg} \) and \( m_3 = 2 \text{ kg} \).
Example

Find the center of mass of the triangle shown below. Assume uniform mass per unit area $\sigma$, i.e. $M = \sigma \cdot (\frac{1}{2} b h)$. 
Center of Mass for System of Objects

\[
\vec{R}_{CM} = \frac{1}{M_{Total}} \sum_{i=1}^{N} M_i \vec{R}_{CM,i}
\]
The disk shown in Case 1 clearly has its CM at the center. Suppose the disk is cut in half and the pieces arranged as shown in Case 2.

In which case is the center of mass highest?

A) Case 1  B) Case 2  C) same
If the total force is zero, the \textit{CM} won’t accelerate.
Three tiny equal-mass magnets are placed on a horizontal frictionless surface at the corners of an equilateral triangle. When the magnets are released, they attract and quickly slide to a single point. What are the coordinates of that point?

\[ X_{CM} \quad Y_{CM} \]

A) 0 0
B) 0 \( H/2 \)
C) 0 \( H/3 \)
D) \( H/4 \) \( H/4 \)
E) \( H/4 \) 0
Flashcard Question

Three tiny equal-mass magnets are placed on a horizontal frictionless surface at the corners of an equilateral triangle. When the magnets are released, they attract and quickly slide to a single point. What are the coordinates of that point?

A) 0 0
B) 0 H/2
C) 0 H/3
D) H/4 H/4
E) H/4 0

Mechanics Lecture 10, Slide 11
Interesting Application: Exoplanet Discovery

- Extrasolar planets usually too small to be seen directly.
- Need to detect indirectly by observing changes to the star.
- “astrometry”: observe “wobble” as star moves in order to keep c.o.m. fixed!
Homework Problem

System of Particles

Four particles are in a 2-D plane with masses, x- and y- positions, and x- and y- velocities as given in the table below:

<table>
<thead>
<tr>
<th></th>
<th>m</th>
<th>x</th>
<th>y</th>
<th>vx</th>
<th>vy</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.8 kg</td>
<td>-2.8 m</td>
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1) What is the x position of the center of mass? m Submit

2) What is the y position of the center of mass? m Submit

\[
x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}
\]

\[
y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4}{m_1 + m_2 + m_3 + m_4}
\]
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3) What is the speed of the center of mass? ___________ m/s

\[
\begin{align*}
\mathbf{v}_{\text{cm},x} &= \frac{m_1 v_{1,x} + m_2 v_{2,x} + m_3 v_{3,x} + m_4 v_{4,x}}{m_1 + m_2 + m_3 + m_4} \\
\mathbf{v}_{\text{cm},y} &= \frac{m_1 v_{1,y} + m_2 v_{2,y} + m_3 v_{3,y} + m_4 v_{4,y}}{m_1 + m_2 + m_3 + m_4} \\
\mathbf{v}_{\text{cm}} &= \sqrt{\mathbf{v}_{\text{cm},x}^2 + \mathbf{v}_{\text{cm},y}^2}
\end{align*}
\]
System of Particles

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4) When a fifth mass is placed at the origin, what happens to the horizontal (x) location of the center of mass?
A. It moves to the right.
B. It moves to the left.
C. It does not move.
D. It can not be determined unless you know the mass.

\[ \text{CM moves toward } m_5 \]
System of Particles

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5) When a fifth mass is placed at the center of mass, what happens to the vertical (y) location of the center of mass?
A ☐ It moves up.
B ☐ It moves down.
C ☐ It does not move.
Two pucks of equal mass, on a frictionless table, are being pulled at different points with equal forces. Which one gets to the end of the table first?

A) Puck 1  B) Puck 2  C) Same
Two pucks of equal mass, on a frictionless table, are being pulled at different points with equal forces. Which one gets to the end of the table first?

1) 
\[ M \quad T \quad a_1 \]

2) 
\[ M \quad T \quad a_2 \]

\[ \vec{a}_{CM} = \frac{F_{Net,External}}{M_{Total}} \quad \text{SAME} \]

C) Same
Two objects, one having twice the mass of the other, are initially at rest. Two forces, one twice as big as the other, act on the objects in opposite directions as shown above.

1) Which of the following statements about the acceleration of the center of mass of the system is true?

- $a = F/M$ to the right
- $a = F/(3M)$ to the right
- $a = 0$
- $a = F/(3M)$ to the left
- $a = F/M$ to the left

Submit
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- $a = \frac{F}{M}$ to the left

System of Two Masses: Question 1 (N = 79)
Two Men Pulling

Two guys who weight the same are holding onto a massless pole while standing on horizontal frictionless ice.

1) If the guy on the left starts to pull on the pole, where do they meet?

-3 m
0
+3 m

Submit
Now, if the guy on the left has mass 2M and the guy on the right 1M, where would they meet?

a) -3 m  b) -1 m  c) 0 m  d) +1 m
Now, if the guy on the left has mass 2M and the guy on the right 1M, where would they meet?

a) -3 m  b) -1 m  c) 0 m  d) +1 m