The Measurement of The Rydberg Constant

By Alex Gibbs
In 1913, Niels Bohr proposed his “Bohr Model”; a working model of the hydrogen atom.

Supported his model by deriving the Rydberg Formula: \( \frac{1}{\lambda} = R \left[ (\frac{1}{n_f^2}) - (\frac{1}{n_i^2}) \right] \)

What is the Rydberg Constant R?
Theoretical Derivation of Rydberg Formula

- Electron contained in orbit around nucleus by a balance between Centripetal and Coulomb Forces:

\[
\left(\frac{1}{4\pi \varepsilon_0}\right)\left(\frac{e^2}{r_n^2}\right) = \frac{m_e v_n^2}{r_n}
\]  \hspace{1cm} (1)

- Angular momentum \( L = m_e v_n r_n \) is quantized and is an integral multiple of \( \frac{\hbar}{2\pi} \) so:

\[
m_e v_n r_n = n\left(\frac{\hbar}{2\pi}\right)
\]  \hspace{1cm} (2)

- Solving Equations (1) and (2) gives:

\[
r_n = \varepsilon_0 \left(\frac{n^2 \hbar^2}{\pi m_e e^2}\right) \quad \text{and} \quad v_n = \left(\frac{1}{\varepsilon_0}\right)\left(\frac{e^2}{2\hbar}\right)
\]
Theory Continued

- In the Bohr Model Total energy is equal to Kinetic energy plus Potential energy:

\[ E_n = KE_n + PE_n = \left( \frac{1}{2} \right) m_e v_n^2 + \left( -\frac{1}{4\pi \varepsilon_0} \right) \left( \frac{e^2}{r_n} \right) = \left( \frac{1}{\varepsilon_0} \right) \left( \frac{m_e e^4}{8n^2 h^2} \right) - \left( \frac{1}{\varepsilon_0} \right) \left( \frac{m_e e^4}{4n^2 h^2} \right) \]

- Which reduces to:

\[ E_n = -\left( \frac{1}{\varepsilon_0} \right) \left( \frac{m_e e^4}{8n^2 h^2} \right) \]

- Since the principle quantum number \( n \) characterizes the orbit, the energy change due to orbit transitions is:

\[ \Delta E = E_i - E_f = \left( \frac{m_e e^4}{8\varepsilon_0^2 h^2} \right) \left[ \left( \frac{1}{n_f^2} \right) - \left( \frac{1}{n_i^2} \right) \right] \]

- Where the Rydberg Constant is:

\[ R_H = \left( \frac{m_e e^4}{8\varepsilon_0^2 h^2} \right) = 1.09 \times 10^7 \text{ m}^{-1} \]
Experimental Methods

- A Hydrogen Discharge lamp is used to excite atoms to make transitions between energy states producing light.
- The light is collimated and sent through a diffraction grating.
- The diffraction grating separates the light into its spectrum.
- A telescope is attached to view the spectrum and measure the angle of diffraction.
Plan of Analysis

- Calibration of apparatus
- Hydrogen Visible Wavelengths Derivation
- Rydberg Constant Derivation
Calibration

- A good calibration is crucial when conducting this experiment with this apparatus.
- Diffraction angles are a “finger print” of the element so must be correct and not distorted.
- Good calibration is achieved when diffraction angles are symmetric about $\theta_f = 0$.
- Calibration is done using mercury lamp of known wavelength: $\lambda$ (green) = 5460.74 Å.
## Calibration Data

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<th>n</th>
<th>Θr(bad)</th>
<th>Θr(ok)</th>
<th>Θr(good)</th>
<th>n*λ (angstoms)</th>
<th>sin[Θr(bad)]</th>
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Determination of Visible Wavelengths

- Measure $\Theta_r$ at $n = \pm 1, \pm 2, \pm 3$ for the visible hydrogen wavelengths H(red), H(blue) and H(purple)
- Make a linear least squares fit from the data pairs
- Extract values of wavelengths from the slopes of the graphs
- $\lambda$(red) = 665 ± 21 nm,
- $\lambda$(blue) = 462 ± 9 nm,
- $\lambda$(purple) = 432 ± 4 nm
Determination of Rydberg Constant

- Make a linear least squares fit of the data pairs:
  \[ \left( \frac{1}{n^2} \right) - \left( \frac{1}{2^2} \right) = \frac{1}{\lambda} = (x, y) \]
- Determine Rydberg Constant from slope.
- Best Fit gives:
  \[ R_H = 1.17 \times 10^7 \pm 0.03 \text{ m}^{-1} \]
Final Result & Discussion

- Ok agreement between experimental and expected data. $x^2 = 2.5$
- Corresponds to a percentage probability of 8.2%.
- Greater than 5% level but not by much.
- 7.5% error relative to theoretical value.
Conclusion

- The hydrogen spectrometer can be used to obtain an ok value of the Rydberg constant,
  \[ R_H = 1.17 \times 10^7 \pm 0.03 \text{ m}^{-1} \]

- Further calibration can lead to a Really Good value for the Rydberg constant: Bad Calibration gives:
  \[ R_H = 1.21 \times 10^7 \pm 0.05 \text{ m}^{-1} \]
  2.4% improvement in accuracy