Relativistic Invariants in Scattering

\[ S = (p_1 + p_2)^2 = (p_3 + p_4)^2 = E_\text{TOT}\^2 \]

\[ t = (p_1 - p_3)^2 = (p_2 - p_4)^2 \]

\[ u = (p_1 - p_4)^2 = (p_2 - p_3)^2 \]

"Mandelstam" VARIABLES

Phys Rev 112 1344 (1958)

=> Now consider an elastic scattering process in which

Particle 1 \rightarrow Particle 3

Particle 2 \rightarrow Particle 4
\[ t = (p_i - p_2) = (E_i^c - E_i^f, \vec{p}_i^c - \vec{p}_i^f)^2 \]

In CCM \[ |\vec{p}_i^c| = |\vec{p}_i^f| = |\vec{p}_m| \]

\[ \therefore E_i^c = E_i^f \]

\[ t = (0, \vec{p}_i^c - \vec{p}_i^f)^2 \]

\[ = - \left( |\vec{p}_i^c|^2 + |\vec{p}_i^f|^2 - 2\vec{p}_i^c \cdot \vec{p}_i^f \right) \]

\[ = -2 |\vec{p}_m|^2 (1 - \cos \theta_{cm}) \quad \text{Negative, \: space-like} \]

**What is the t-channel process?**

\[ X \]

\[ \begin{array}{c}
1 \\
\downarrow \\
M = \sqrt{s} \\
\downarrow \\
2 \\
\downarrow \\
4 \\
\downarrow \\
t
\end{array} \]

Let \( q^2 = -t \) \( q = \text{"momentum transfer"} \)

\[ q^2 = 2P_m^2 (1 - \cos \theta_{cm}) \]

\[ dq^2 = -2P_m^2 d(\cos \theta_{cm}) \]

\[ \frac{d(\cos \theta)}{dq^2} = \frac{-d\Omega}{2}\frac{d\theta}{d\Omega} \]
Rutherford scattering \[ p = mv \] (Nuclear Physics)

\[ m \ll m_{\text{target}} \] (Fixed target, Lab = CM)

\[
\frac{dc}{d^2} = \left( \frac{Z^2 e^2}{4\pi} \right)^2 \frac{1}{S_{\text{nu}} G \gamma^2}
\]

\[
\frac{1}{\sqrt{(mv)^2}} \frac{dc}{dq^2} = \left( \frac{Z^2 e^2}{2mv^2} \right)^2 \frac{1}{\left(1 - \cos \theta \right)^2}
\]

\[ S_{\text{nu}} \frac{x}{z} = \sqrt{1 - \cos \theta} \]

\[ q^2 = 2 \rho_{\text{cm}}^2 (1 - \cos \theta_{\text{cm}}) \]

\[ q^4 = \frac{m_{\text{nu}}^4}{2(1 - \cos \theta)} \]

\[
\frac{dc}{dq^2} = \frac{2}{m v^2} \left( \frac{Z^2 e^2}{2mv^2} \right)^2 \frac{16m_{\text{nu}}^4}{q^4}
\]

\[
\frac{dc}{dq^2} = \frac{4\pi (Z^2 e^2)^2}{v^2} \frac{1}{q^4}
\]

**Rutherford cross section in terms of momentum transfer in c.m. system.**

\[ U - \text{chiral?} \]

\[ U = (p - p_1) \]

**Cubic in cm for MW**
Observations

⇒ Positive charge & mass of atomic nucleus concentrated at center (Rutherford scattering)

⇒ Mass of nucleus \( > Z \times m_p \) (How do we know?)

⇒ Neutron discovered 1932

\[ \text{Po} \xrightarrow{\alpha} \text{Be} \text{ } \text{unknown neutral, penetrating radiation} \xrightarrow{\rho^+} \]

"Neutron" w/mass \( m_p \) was suggested explanation for high-energy

⇒ ~1932 Nucleus consists of some protons & neutrons held together by stronger-than-electric force.
Mass of Nucleus

\[
\text{Nucleus} = \text{proton or neutron}
\]

\[
A \times Z
\]

His mass \( M_{AZ} < Zm_p + (A-Z)m_n \)

Difference is \underline{Binding Energy}

\[
BE = \sum M_{AZ} - [Zm_p + (A-Z)m_n] \frac{2}{3} \times c^2
\]

- Negative #
- \(-1 \times BE\) read to unbind nucleus

\[
\frac{B}{A} = -\frac{BE}{A}
\]

"Binding energy per nucleon"

\[
\text{Fig 2.1 of text; } \frac{B}{A} \text{ vs } A \text{ very important graph}
\]

\[
\Rightarrow \text{Before we get to that, let's ask how do we know the size of the atomic nucleus?}
\]