Lecture 16: Nuclear Physics Review

Midterms:
The delicious ice cream center of your semester.

Physics 5110
Spring Semester 2015
Midterm Exam

• Midterm on Monday March 2\textsuperscript{nd}

• Allowed resources:
  – One sheet of paper with notes, equations
  – Calculator, pencil or pen

• Subject matter: Nuclear physics
  – Short work out problems
  – Short answer

• Study homeworks, lecture examples, practice exams.
Chapters/Topics

- Rutherford Scattering
- Nuclear Phenomenology
- Nuclear Models (liquid drop, shell)
- Radiation ($\alpha,\beta$)
- Applications (fission, fusion, decay)
Ch 1: Rutherford Scattering

- Differential cross section $d\sigma/d\Omega$
- Rutherford $d\sigma/d\Omega$
- Possible short answer questions
  - What is a differential cross section?
  - Explain how Rutherford scattering demonstrates that there is a nucleus.
  - What are Mandelstam variables $s,t,u$, and what do they signify physically?
6. (22 pts) A beam of $6.00 \times 10^5$ 10 MeV $\alpha$ particles per second is incident on a 0.25 cm thick silver (Ag) foil.

(a) (8) The density of silver is 9.320 g/cm$^3$. What is the number of target nuclei per unit area in the foil?

(b) (14) At what maximum distance from the foil may I place a 2 cm$^2$ detector at an angle of 22$^\circ$ from the incident beam, if I wish to count at least 20 $\alpha$-particles per second? In MKS units, $\epsilon_0 = 8.854 \times 10^{-12}$ C$^2$/(N·m$^2$).
Result from Lecture 2

\[ dN_{\text{hits}} = \frac{N_0 \rho t A_0}{A} \left( \frac{d\sigma}{d\Omega} \right) d\Omega \]

- \( N_0 \) = number of beam particles
- \( \rho, t \) = target density, thickness
- \( A_0, A \) = Avogadro's #, Atomic mass

Hits recorded by detector

Solid angle of detector

Differential Cross Section (physics)
Classical Scattering Theory (Fixed Target)

\[
\frac{d\sigma}{d\Omega} = \frac{-2\pi b db}{2\pi \sin \theta d\theta} = \frac{-b}{\sin \theta} \frac{db}{d\theta}
\]
Do the math...

- And after some arithmetic (homework)

\[
\frac{d\sigma}{d\Omega} = \frac{-b}{\sin \theta} \frac{db}{d\theta}
\]

\[
= \left( \frac{zz'e^2}{4E} \right)^2 \text{csc}^4 \left( \frac{\theta}{2} \right)
\]

\[
= \left( \frac{zz'e^2}{4E} \right)^2 \frac{1}{\sin^4 \left( \frac{\theta}{2} \right)}
\]
Ch 2: Nuclear Phenomenology

• Nuclear mass, size, spin, stability.
• How do we know the above?
• Possible short answer questions
  – What is a form factor?
  – Explain how we can know the size of an atomic nucleus.
  – What is binding energy, how is it determined?
1. (16 pts) Describe a scattering experiment to determine the radius of a gold (\(^{197}\text{Au}^{79}\)) nucleus. In particular, answer the following:

(a) (4) Given that the radius of a single nucleon is about \(1.2 \times 10^{-15}\) m, what value do you expect for the radius of a gold nucleus?

(b) (4) What momentum should the scattering probe have, and why?

(c) (4) What particle would you use as the scattering probe, and why?

(d) (4) Explain the inadequacies of the Rutherford model in describing this scattering experiment, and how the Rutherford model should be modified for this particular situation.
Size of Nucleus

- Method suggested by diffraction of light
- Scatter particles of $\lambda = h/p$ off of nucleus
Fig. 3.3 The square of the form factor $|F(q^2)|^2$ as a function of $q$ for a model I nucleus having $a = 4.1$ fm. The abscissa is also marked in inverse fermis ($q/\hbar$) and in degrees for an angle of scatter at a fixed nucleus for incident electrons of 450 MeV. Note that the ordinate is logarithmic.
Ch 3: Nuclear Models

- Liquid Drop Model
- Shell Model

- Possible short answer questions
  - Describe origin of term(s) in semi-empirical mass formula
  - Describe role of spin-orbit interaction in the shell model
Goal of Nuclear Models

- Can we model this curve?
- Can we make other predictions?
The Semi-Empirical Mass Formula (Bethe-Weizaecker)

\[
\text{Binding Energy} = -a_v A + a_s A^{2/3} + a_c \frac{Z^2}{A^{1/3}} + a_a \frac{(N - Z)^2}{A} \pm a_p \frac{1}{A^{3/4}}
\]

volume  \[\downarrow\]  Coulomb  \[\downarrow\]  surface  \[\downarrow\]  asymmetry  \[\downarrow\]  pairing
4. (20 pts) Consider the 3 states consisting of 30 nucleons, $^{30}P^{15}$, $^{30}Si^{14}$, and $^{30}Al^{13}$.

(a) (12) Using the semi-empirical mass formula, calculate which of these three states is the most stable. Show all work. Note that you do not have to calculate the mass or binding energy of each state, only quantitatively determine which is the most stable. Explain your choice of the most stable state. Does it agree with what is observed?

<table>
<thead>
<tr>
<th>$a_V$</th>
<th>15.56 MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_S$</td>
<td>17.23 MeV</td>
</tr>
<tr>
<td>$a_C$</td>
<td>0.697 MeV</td>
</tr>
<tr>
<td>$a_A$</td>
<td>23.285 MeV</td>
</tr>
<tr>
<td>$a_P$</td>
<td>12.0 MeV</td>
</tr>
</tbody>
</table>

(b) (8) Draw the “arrow” diagrams for the unstable states, indicating all particles and nuclei participating in the decays.
“Pairing Term”

\[
\text{Binding Energy} = -a_v A + a_s A^{2/3} + a_c \frac{Z^2}{A^{1/3}} + a_a \frac{(N - Z)^2}{A} \pm a_p \frac{1}{A^{3/4}}
\]

- PP and NN are more strongly bound than PN.
- Introduce a term that has
  - Greater BE for $Z_{\text{even}} N_{\text{even}}$.
  - Zero BE for $Z_{\text{even}} N_{\text{odd}}$ or $Z_{\text{odd}} N_{\text{even}}$.
  - Less BE for $Z_{\text{odd}} N_{\text{odd}}$.

Phenomenological A dependence
3. (10 pts) Explain what is meant by the *spin-orbit interaction*. What problem was solved by inclusion of the spin-orbit interaction into the shell model of the nucleus, and how?
Spin Orbit Effect in Shell Model

So the total splitting is given by:

\[ E_{n,l}(j = l - 1/2) - E_{n,l}(j = l + 1/2) = \hbar^2 (l + 1/2) \int \Psi^*_n(l, \vec{r}) \Psi_n(l, \vec{r}) f(r) d^3 \vec{r} \]

→ splitting increases for larger \( l \)
→ can produce “level crossing”
→ hence desired shell structure!
Fig. 3.4 Energy levels in a single-particle shell model. The boxed integers correspond to the magic nuclear numbers.
**Unpaired nucleons** provide spin-parity assignment:

- Even-even nuclei → have zero spin (observed)
- Odd-Odd nuclei → can't predict :-( (we don't know how N-P will pair)
- Odd-Even (i.e. odd A) nuclei
  - Determine which type of nucleons is odd-numbered
  - Count odd nucleons to determine filled shells
  - Spin-parity of last nucleon is that of nucleus
Examples: Use shell model to estimate spin-parity and magnetic dipole moment of...

- Tritium $^3\text{H}^1$
- Lithium $^7\text{Li}^3$
- Carbon $^{11}\text{C}^6$
Ch 4: Nuclear Radiation

- Alpha, Beta, Gamma Decays
- Possible short answer questions
  - Describe the role of “tunneling” in nuclear decays.
  - Differentiate between the three types of “beta” decay.
  - Describe how we infer the existence of neutrinos from beta decay
5. (22 pts) Americium ($^{241}\text{Am}^{95}$) — used in household smoke detectors — undergoes $\alpha$-decay to $^{237}\text{Np}^{93}$ with a half-life of 432.2 years. When bombarded with neutrons, $^{241}\text{Am}^{95}$ can become $^{242}\text{Am}^{95}$, which $\beta^-$ decays to $^{242}\text{Cm}^{96}$ with a half-life of 16.02 hours.

(a) (8) An $^{241}\text{Am}^{95}$ smoke detector works by observing fluctuations in the rate at which alpha particles are detected, when the alpha particles are absorbed by smoke. How long will it take (in years) until a smoke detector sees a 10% change in the alpha rate due to decay of the source?

(b) (14) A sample of $^{242}\text{Am}^{95}$ is observed with a Geiger Counter covering a solid angle of $4\pi \times 10^{-3}$ Steradians. The Geiger Counter records $\beta^-$ particles (electrons) at a rate of 100.0 Hz. If the sample is currently 4.25 hours old, how many $^{242}\text{Am}^{95}$ atoms were in the sample initially?
Ch 5: Applications of Nuclear Physics

- Fission
- Fusion
- Radioactive decay law
- Possible short answer questions
  - What is difference between spontaneous and induced fission?
  - Interpret “Neutrons in Natural Uranium” plot.
  - Explain the “moderator” in fission reactors.
  - Why do we fuse light nuclei and “fiss” heavy nuclei to produce energy?
Neutron interactions in Natural Uranium
2. (10 pts) Explain the mechanism of *induced fission*. Why is induced fission a desirable energy source in a controlled nuclear reaction? What type of nuclei are typically employed in induced fission reactors, and why?