Remainder of Course

- 4/22 – Standard Model; Strong Interaction
- 4/24 – Standard Model; Weak Interaction
- 4/27 – Course review
- 5/01 – Final Exam, 3:30 – 5:30 PM
  - Practice Final on Course Web Page
  - See HW #12 (not to be collected) on course web page, for quark diagram practice. This is testable.
Toy Model Decay Diagrams

First order

Next order
Toy Model Scattering Diagrams
The Real World: Quantum Electrodynamics

Primitive Vertex
Space-time/Feynman Diagram Summary

- Diagrams should not be interpreted as the literal trajectories of particles
- Nevertheless, they convey information about the interactions of quantum fields
  - Represent components of matrix elements
  - Provide “cartoon” means of understanding fundamental interactions
- Calculation of matrix elements for all forces is beyond the scope of 5110
- Being able to write down all the diagrams is not!
Deep Inelastic Scattering Confirmed Reality of **Quarks**

- **Hadrons**
  - Mesons ($q\bar{q}$)
  - Baryons ($qqq$)

- More details needed on nature of force between quarks.

- Clue: Existence of **ground states** consisting of **3 identical quarks**

\[
\Delta^{++} \quad J = \frac{3}{2} \quad I = \frac{3}{2} \quad uuu
\]

\[
\Delta^{-} \quad J = \frac{3}{2} \quad I = -\frac{3}{2} \quad ddd
\]

\[
\Omega^{-} \quad J = \frac{3}{2} \quad I = 0 \quad sss
\]

\[\text{no orbital A.M... ground state!}\]
Problem: $u^\uparrow u^\uparrow u^\uparrow$ is three identical fermions in same quantum state!

- There has to be an additional internal quantum number
- “Color”; red, green or blue
- Never observed before because all hadrons are “color neutral” (like atoms are “charge neutral”).

- Two types of color neutrality:
  - red + red = black
  - red + blue + green = white
Quantum Chromodynamics

- Color is “charge” of strong interaction
- “Gluon” is exchange particle
- Primitive vertices

Consequences
- Quarks cannot be free
- “Glueballs” possible (though not yet observed)

QCD Feynman Rules, see e.g. Griffiths
Example: Draw a quark/gluon level Feynman diagram for the strong interaction process...

\[
K^- + p \rightarrow \Omega^- + K^+ + K^0
\]

\[
\bar{u}s + uud \rightarrow sss + \bar{u}s + d\bar{s}
\]
Standard Model:
Quarks and the Weak Interaction
Weak Primitive Vertices

\[ q' \rightarrow \quad w^\pm \quad \rightarrow \quad q \]

\[ v_L \rightarrow \quad w^\pm \quad \rightarrow \quad L \]
Try these...

- $\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e$
- $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$
- $K^0 \rightarrow \pi^- + e^+ + \nu_e$
- $p + e^- \rightarrow n + \nu_e$
- $\Omega^- \rightarrow \Xi^0 + \pi^-$
Bremsstrahlung

Pair Production
Additional Weak Primitive Vertices ("Neutral Current")
Note: There are no flavor changing neutral currents
“Charged Current”

```
q' \rightarrow \rightarrow W^\pm \rightarrow q
```

“Neutral Current”

```
L \rightarrow \rightarrow Z^0 \rightarrow L
q \rightarrow \rightarrow Z^0 \rightarrow q
L \rightarrow \rightarrow L
v_L \rightarrow \rightarrow Z^0 \rightarrow v_L
```
Strong Primitive Vertices
Electromagnetism Primitive Vertices
A last few topics...
The fourth ("charmed") quark

\[ K^+ \rightarrow \mu^+ \nu_\mu \quad \text{BF} = 0.6343 \]
\[ \tau_+ = 1.24 \times 10^{-8} \text{ s} \]
\[ \tau(K^+ \rightarrow \mu^+ \nu_\mu) = 1.95 \times 10^{-8} \text{ s} \]

\[ K^0_L \rightarrow \mu^+ \mu^- \quad \text{BF} = 7.27 \times 10^{-9} \]
\[ \tau_L = 5.18 \times 10^{-8} \text{ s} \]
\[ \tau(K^0_L \rightarrow \mu^+ \mu^-) = 7.13 \text{ s} \]

Note space horizontal, time vertical

Doesn't happen. No flavor-changing neutral currents
The fourth ("charmed") quark

What about...

- This process has two quark vertices, therefore reduced amplitude.
- But not enough for $10^{-8}$ suppression!
The fourth ("charmed") quark

- What if there were four quarks? (G.I.M., 1970)

- Contributions from these two diagrams cancel.
- Explained $K_L \rightarrow \mu^+\mu^-$ suppression
- Charmed quark observed 1974
Quark Mixing

• “Weak” d, s quarks are mixtures of “strong” quarks:

\[ d_w = \alpha d_s + \beta s_s \]
\[ s_w = \gamma d_s + \delta s_s \]

\[
\begin{pmatrix}
  d_w \\
  s_w
\end{pmatrix}
= \begin{pmatrix}
  \alpha & \beta \\
  \gamma & \delta
\end{pmatrix}
\begin{pmatrix}
  d_s \\
  s_s
\end{pmatrix}
\]

• 2x2 matrix is known as “Cabibbo” or “quark mixing” matrix C.

• Conservation of probability requires that C be “unitary”, i.e. \( C^\dagger C = 1 \)
Quark Mixing

Thus C can be written as a rotation matrix

\[
\begin{pmatrix}
  d_w \\
  s_w \\
\end{pmatrix}
= \begin{pmatrix}
  \cos \theta_c & \sin \theta_c \\
  -\sin \theta_c & \cos \theta_c \\
\end{pmatrix}
\begin{pmatrix}
  d_s \\
  s_s \\
\end{pmatrix}
\]

\[
d_w = d_s \cos \theta_c + s_s \sin \theta_c
\]

\[
s_w = -d_s \sin \theta_c + s_s \cos \theta_c
\]
Quark Mixing

- First evidence for 3\textsuperscript{rd} quark generation...
- Cabibbo scheme generalizes to 3x3 matrix (Cabibbo-Kobayashi-Maskawa or CKM)

\[
\begin{pmatrix}
  d_w \\
  s_w \\
  b_w 
\end{pmatrix} = \begin{pmatrix}
  V_{ud} & V_{us} & V_{ub} \\
  V_{cd} & V_{cs} & V_{cb} \\
  V_{td} & V_{ts} & V_{tb} 
\end{pmatrix} \begin{pmatrix}
  d_s \\
  s_s \\
  b_s 
\end{pmatrix}
\]
Quark Mixing

• In general, to be unitary a 3x3 matrix must have 4 independent parameters; three real angles and one complex phase.

\[
\begin{pmatrix}
  d_w \\
  s_w \\
  b_w
\end{pmatrix} = 
\begin{pmatrix}
  V_{ud} & V_{us} & V_{ub} \\
  V_{cd} & V_{cs} & V_{cb} \\
  V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
  d_s \\
  s_s \\
  b_s
\end{pmatrix}
\]

\[
\begin{bmatrix}
  1 & 0 & 0 \\
  0 & c_{23} & s_{23} \\
  0 & -s_{23} & c_{23}
\end{bmatrix}
\begin{bmatrix}
  c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\
  0 & 1 & 0 \\
  -s_{13}e^{i\delta_{13}} & 0 & c_{13}
\end{bmatrix}
\begin{bmatrix}
  c_{12} & s_{12} & 0 \\
  -s_{12} & c_{12} & 0 \\
  0 & 0 & 1
\end{bmatrix}
\]
Quark Mixing

• In general, to be unitary a 3x3 matrix must have 4 independent parameters; three real angles and one complex phase. In general
  – Complex phase is origin of CP violation
  – \( \delta \) is the reason we're here!
Unification of Forces

- EM and weak forces have been shown to be different aspects of the same force ("electroweak"). Their coupling constants can be related by...

\[ \sqrt{2} G_F = \frac{\pi \alpha_{EM}}{m_w^2 (1 - m_w^2/m_Z^2)} \]

- Assuming that the strong force can be unified with electroweak, \( \alpha_{EM} \) and \( \alpha_S \) will also be related.
Unification of Forces

- At high energies, ball in “sombrero” potential has no preferred orientation
- At low energies, symmetry of system is broken

\[ V(x, y) = -\frac{1}{2} m\omega^2 (x^2 + y^2) + \frac{\lambda}{4} (x^2 + y^2)^2 \]
Unification of Forces

Normal modes of sombrero potential

- Motion along valley floor: requires no energy.
- Motion perpendicular to valley floor: requires energy
Unification of Forces

- Sombrero potential represents **Higgs Field**
- Sphere represents EW force carrier
- At low energies, force carrier has normal modes corresponding to
  - massless (zero energy $\rightarrow$ photon)
  - massive (nonzero energy $\rightarrow$ $W^{\pm}, Z^0$)
Standard Model Couplings

- quarks
- gluons
- photon
- Higgs
- W boson
- Z boson
- charged leptons
- neutrinos