LECTURE 3: LINEAR DIFFERENTIAL OPERATIONAL AMPLIFIERS

Operational Amplifier

The differential operational amplifier has characteristics which, at audio frequencies or less (below about 20,000 Hz), approach those of the ideal amplifier. (Hz or "Hertz" is the proper symbol for cycles per second.) The actual device we will use in the laboratory will be the LF351, an excellent device which sells for under $1.00. (You couldn't build an equivalent device for anywhere near the price.) We will be assuming some "ideal" characteristics; later we will study the limitations. We will use the jargon-term "op amp" to mean differential operational amplifier.

The ideal characteristics we will assume will include infinite input resistance, zero output resistance and "nearly infinite" voltage gain. The symbol for an operational amplifier is:

\[\text{INVERTING INPUT} \quad \triangle \quad \text{OUTPUT} \quad \text{NON-INVERTING INPUT} \]

Usually only these active circuit terminals are shown. It is assumed that power is supplied to other appropriate terminals (typically +15 V and -15 V though in lab we use +12 V and -12 V).

\[V_- \quad - \quad +V_{CC} \quad -V_{EE} \quad V_+ \quad + \quad V_o \]

\(V_{CC}\) is the usual name for the positive supply, and \(-V_{EE}\) is the usual name for the negative supply. Both supply voltages, \(V_-\), \(V_+\), and \(V_o\) are measured relative to ground. A note on a convention used in this diagram is in order.

\[\text{___} \] indicates two wires crossing without connection.

\[\text{___} \] or \[\text{___} \] indicates wires which are connected together.
Many op amps (but not the LF351) require one or two external capacitors and possibly a resistor to prevent oscillations. Such devices are called "uncompensated" op amps. Specification sheets for the LF351 are included in Appendix Ic.

The basic characteristic of the ideal differential operational amplifier can be expressed in the following terms. If \( V_+ \) is the signal applied to the non-inverting input, \( V_- \) is the signal applied to the inverting input, and \( V_0 \) is the output signal:

\[
V_0 = A_{VD}(V_+ - V_-)
\]  
(1)

With minor adjustments, equation (1) is valid on a D.C. basis as well as for small signal changes. In the specifications for the LF351, \( A_{VD} \) is seen to be typically 100,000. If, for example, you were amplifying the signal voltage between your two wrists for an electrocardiogram, you would have \( V_+ - V_- \approx 3 \text{ mV}(3 \times 10^{-3} \text{ V}) \). An amplification of 100,000 in Eq. (1) would give you an output of 300 V. With a ±12 volt power supply, you cannot get 300 volts. In fact, \( V_0 \) can only range from about +10 V to -10 V. When the output reaches these levels we say the device is "saturated." Between these limits we say the device is operating in its "active region." If we exaggerate the region of inputs over which the device is active we can plot a "transfer function" of the device. (A transfer function is simply the relationship between some input and the resulting output.)

\[
\begin{align*}
V_+ - V_- & \quad \text{Saturation Region} \\
-V_{sat} & \quad \text{Saturation Region} \\
-V_{EE} & \\
\end{align*}
\]

\[
\begin{align*}
& \quad V_0 \quad +V_{CC} \\
& \quad +V_{sat} \\
& \quad V_+ - V_- \\
& \quad \text{active region} \\
\end{align*}
\]

So in our electrocardiogram example the amplifier will simply saturate at levels near plus and minus 12 volts and you would lose knowledge of the waveform. Clearly, we need some way to reduce the gain of the amplifier. One way would be to put a 300 to 1 voltage divider on the input:
If we want a one volt output, this procedure will work if the gain of the op amp is really 100,000. According to the specification sheet, it might be as low as 25,000 in which case we would get 0.25 V at the output. No upper limit to the gain is given so we might have an op amp that would saturate despite our 300 to 1 voltage divider on the input. (Furthermore, our input signal at the amplifier would be about 10 μV which might be comparable to the noise of a real, nonideal op amp.) Clearly, we need a better approach. The solution is to use "negative feedback."

**Negative Feedback**

Let us now consider a signal which is referenced to ground. (In the previous example, we could ground one wrist and regard the other wrist as our signal source.) The trick is to connect the signal to the non-inverting input and connect a fraction of the output, via a voltage divider, to the inverting input. The feedback is said to be "negative" because an increase in $V_o$ causes an increase in $V$ which has the effect of decreasing $V_o$. Thus "negative" feedback tends to be stabilizing. Let us do the algebra and see what we get for the gain.

![Diagram](image)

$$V_o = A_{VD}(V_+ - V_-)$$

But:

$$V_+ = V_S \text{ (signal voltage)} \text{ and } V_- = V_oR_1/(R_1 + R_2)$$

Note that for $V_o$ we have used the fact that the amplifier has a high input resistance; it does not load the voltage divider. Making the substitutions:

$$V_o = A_{VD}[V_S - V_oR_1/(R_1 + R_2)]$$  \hspace{1cm} (2)
Exercise 1

Solve for $V_o$ and show that if $A_{VD} R_1/(R_1 + R_2) \gg 1$ then:

$$V_o/V_s = (R_1 + R_2)/R_1$$  \hspace{1cm} (3)

We call $ACL = V_o/V_s$ the "closed-loop gain of the non-inverting configuration". Note that the criterion for equation (3) is that $A_{VD} \gg ACL$. Note also that, under the assumptions made, $ACL$ is a function only of resistor values. (Remember that resistors are "the cheapest, most dependable, and most precise elements available to the electronics circuit designer," and that the circuit designer should "make them the critical elements in circuits whenever possible.")

In the example we have been considering, we wanted a closed loop gain of about 300. Since 300 is much less than even the minimum gain of the LF351, the gain will depend only on the values of $R_1$ and $R_2$ and would not change if, for instance, we substituted a minimum-gain op amp for a high-gain op amp.

Exercise 2

Derive the gain for the inverting configuration:

$$V_s \quad \begin{array}{c} \text{A} \\ \text{in} \end{array} \quad R_1 \quad R_2 \quad \begin{array}{c} \text{Pin} \\ \text{out} \end{array} \quad V_o$$

Ans. $V_o = -R_2/V_1$.

Take the limit in which $|A_{VD}| \gg |ACL|$. Hint: Unless you have unusual insight, you will probably find that, for this problem, the equivalent to equation (2) is a system of 2 or 3 linear equations. Second Hint: For the ideal op amp with infinite input resistance, whatever current flows through $R_1$ also flows through $R_2$.

Input Resistance to Inverting Amplifier

The input resistance at point A is $V_s/I_{in}$. We can get $I_{in}$ from the following:

$$I_{in} = \frac{V_s - V_o}{R_1 + R_2} = \frac{V_s + (R_2/R_1)V_o}{R_1 + R_2} = \frac{V_s(1+R_2/R_1)}{R_1 + R_2} = \frac{V_s}{R_1}$$
\[ R_{in} = \frac{V_S}{I_{in}} = \frac{V_S}{V_S/R_1} = R_1 \quad (R_{in} \text{ for the circuit}) \]

Thus, even if the input resistance of the op amp itself is infinite, the input resistance of the inverting configuration is \( R_1 \).

**Limitations**

Several points should be considered in the design of an op amp circuit. First of all, there are frequency limitations, but we will consider them later. Next, we must consider input and output resistances. The input limitations need not be very serious. The input resistance of the LF351 is typically \( 5 \times 10^{11} \, \Omega \), but for another op amp it might be lower, say 300,000 \( \Omega \). Since the output of the op amp must supply current for \( R_2 \), we must choose \( R_2 \) large enough that the current flowing through it is small compared with the maximum the device can provide. For example, the LF351 typically drives 10 mA (see Positive Current Limit and Negative Current Limit curves). There might be 10 volts across \( R_2 \), so \( R_2 \) would have to be at least 1 k\( \Omega \). If the op amp were to supply current to a load as well as to \( R_2 \), then \( R_2 \) would have to be larger than 1 k\( \Omega \).