FORMULÆ

\[ I = \epsilon_0 c \langle E^2 \rangle_T = \frac{1}{2} \epsilon_0 c E_0^2 \]

\[ \mathcal{E}_\gamma = h \nu_\gamma \]

\[ n = c / v \]

\[ v = \lambda \nu \]

\[ (v = \text{“vee”}; \; \nu = \text{“nu”}. ) \]

\[ k = 2\pi / \lambda \]

\[ \omega = 2\pi \nu \]

\[ \tau = 1 / \nu \]

\[ D = \frac{1}{f} = (n_l - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]

\[ \frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \]

\[ M_T = \frac{y_i}{y_o} = \frac{s_i}{s_o} \]

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

\[ \theta_r = \theta_i \]
Problem 1 (25 points)
A laser beam strikes a slab of glass of thickness 4.0 cm at an angle $\theta_i = 0.20$ radians with respect to the normal. Outside the glass, the speed of light is nearly equal to $c$, the speed of light in vacuum. Inside the glass, the speed of the laser light is $(5/8)c$. The configuration is shown in the diagram below. You may assume that the angle $\theta_i$ is small enough that the “small angle” trigonometric approximation applies. Ignoring any reflected components, how far from the dashed line (the continuation of the normal to the surface at the point of incidence) does the laser beam emerge from the glass? Express your answer numerically to two significant digits.

(Those are “vees”, not “nus”, in the diagram. Apologies for the typeface.)
Problem 2 (25 points)
A laser with power $P$ and wavelength $\lambda$ has an initial circular collimated beam cross section of diameter $D$. It is shone through a Galilean beam-expander composed of a negative (diverging) lens of focal length $-f$ and a positive lens of focal length $4f$, which are separated by a distance $3f$ so that the emerging rays are parallel. The setup is illustrated in the following diagram:

![Diagram](image)

Provide answers to the following three questions in terms of $P$, $\lambda$, $D$, $f$, and fundamental constants.

(a) What is the electromagnetic energy density within the beam after it emerges from the expander? (Hint: work out the irradiance, and use the fact that the beam is traveling at the speed of light $c$.)

(b) What is the amplitude of oscillation of the electric field within the beam after it emerges from the expander?

(c) What is the flux of photons per unit area per unit time through a small surface perpendicular to the beam after it emerges from the expander?
Problem 3 (25 points)

An object $O$ is positioned along the optical axis at a distance of 30 cm from a positive thin lens $A$ with focal length $f_A = 10 \text{ cm}$. A second positive thin lens $B$, with focal length $f_B = 90 \text{ cm}$, is positioned a distance 60 cm from lens $A$ along the optical axis in the direction away from $O$. The configuration is illustrated in the diagram below. Assume paraxial optics throughout this problem. Give quantitative answers as numerical values.

(a) Determine the location of the image of the object formed by lens $A$. Construct a ray diagram showing the location and the height of this image, using the figure below. Indicate whether this image is real or virtual, and whether it is right-side up or inverted. What is the transverse magnification of this image?

(b) Take the image formed by lens $A$ as the object for lens $B$, and determine the location of the image formed by lens $B$. Construct a ray diagram showing the location and height of this image, also using the figure below. Indicate whether this image is real or virtual, and whether it is right-side up or inverted relative to the image formed by lens $A$. What is the transverse magnification of this image relative to the image formed by lens $A$?

(c) What is the overall transverse magnification of the two-lens combination? What does the object look like when seen through this configuration from the right?
Question 1: (5 points) Which of the following is not one of the 5 primary (Seidel) aberrations of third-order optical theory?

(a) Spherical aberration  (c) Astigmatism  (e) Distortion
(b) Coma  (d) Chromatic aberration  (f) Field curvature

Question 2: (5 points) What are the two key components of an optical fiber (labeled 1 and 2 in the diagram below), how do they differ most importantly from one another, and how does the fiber confine and transmit light as a result? (Words are fine; no derivations needed.)

Question 3: (5 points) Match each eye condition to the type of lens with which it is corrected:

(a) Farsightedness  (i) Toric or cylindrical lenses
(b) Nearsightedness  (ii) Positive lenses
(c) Astigmatism  (iii) Negative lenses

Question 4: (5 points) Which of the following are legitimate electric-field components of a harmonic electromagnetic plane wave in free space? (Assume in each case that the magnetic-field component would be as necessary to satisfy the Maxwell equations.)

(a) \( \mathbf{E}(x, y, z, t) = \hat{y}E_0 \sin[k(y - vt)] \)  
(b) \( \mathbf{E}(x, y, z, t) = \hat{y}E_0 \cos[k(ax^2 - vt)] \)  
(c) \( \mathbf{E}(x, y, z, t) = \hat{x}E_o \sin(kz + \omega t) \)  
(d) \( \mathbf{E}(x, y, z, t) = \hat{x}E_o \sin(kz + \omega t) \)  
(e) \( \mathbf{E}(x, y, z, t) = \hat{z}E_o \cos[k(x + vt)] \)

Question 5: (5 points) The diagram below represents an achromatic doublet composed of two different glasses, with \( n_2 > n_1 \). Assume that it has been designed to cancel chromatic aberration between red and blue light for objects at the “X”. Trace (and label) the paths of red and blue rays from the “X” as they traverse the lens and are focused on the right side. (Assume the “X” is located more than one focal length from the lens.)