Only pen or pencil are allowed. No calculators or additional materials.
Problem 1 (25 points)
A harmonic plane wave in vacuum with \( k = \frac{2\pi}{\lambda} \) and \( \lambda = 600 \text{ nm} \) has the form

\[
E(x, y, z, t) = \hat{x}E_0 \sin(kz - \omega t) + \hat{y}E_0 \sin(kz - \omega t).
\]

Write an expression for the wave after it passes through the following three materials (not in sequence—consider this same waveform as the input to all three materials). You may ignore any global phase offsets in your answers.

(a) An HN32 polaroid parallel to the \( x-y \) plane, with its axis in the \( y \) direction. (Recall that an HNXX polaroid transmits XX\% of the total incident irradiance, but none of the irradiance associated with electric field oscillations perpendicular to its axis. Also recall that \( I \propto \langle |E|^2 \rangle_T \).)

(b) A quarter-wave plate (as rated for 600 nm) parallel to the \( x-y \) plane, with its “fast axis” in the \( x \) direction (assume negligible attenuation within the plate.)

(c) A path of 9 cm through a solution of optically active sugar with a specific rotatory power of 5° per cm at \( \lambda = 600 \text{ nm} \). (You may assume either “handedness” of rotation, and assume negligible attenuation within the solution. Please simplify your answer as much as possible.)
Problem 2 (25 points)
A double-slit configuration with known slit separation $a = 0.16 \text{ mm}$ and unknown slit width $b$ and (as diagrammed schematically at left below) is illuminated from one side by coherent monochromatic light. The irradiance distribution seen on a screen located 125 centimeters away from the slits is shown at right below (note units of millimeters in plot).

Using the result that the far-field irradiance distribution is described by

$$I(\theta) = 4I_0 \left( \frac{\sin \beta}{\beta} \right)^2 \cos^2 \alpha$$

where

$$\beta = \frac{\pi b}{\lambda} \sin \theta \quad \text{and} \quad \alpha = \frac{\pi a}{\lambda} \sin \theta,$$

determine the values of the illuminating wavelength $\lambda$ and the slit width $b$. (You may use the small-angle approximation of $\sin \theta \approx \tan \theta \approx \theta$.)
Problem 3 (25 points)
The bottom side of a compact disc (CD) is illuminated with normally incident white light from a flashlight. A white-light image is observed in reflection directly back from the surface of the CD. Offset by a relatively small angle from this, a reflected spectrum is seen. At a distance of about 1.0 meter from the CD, an observer finds that the eye must be moved through a perpendicular distance of 15 centimeters in order to shift between the bluest visible wavelength (about 390 nm) and the reddest visible wavelength (about 750 nm) in the spectrum, relative to the center of the observer’s field of view. The situation is diagrammed schematically below.

Using the grating equation
\[ a(\sin \theta_m - \sin \theta_i) = m\lambda \]
and the small-angle approximation, find the spacing \( a \) between the grooves of the CD according to the values reported for this “experiment”.

![Diagram of the CD setup](image)
Question 1: (5 points) Which of the following phenomena can convert natural light propagating in air into plane-polarized (or partially plane-polarized) light?

(a) scattering  (c) water reflection at normal incidence  (e) birefringent materials
(b) optical activity  (d) water reflection at oblique incidence  (f) dichroic materials

Question 2: (5 points) A single-layer anti-reflective coating works by controlling the relative phase and relative amplitude of the waves reflected off the front and back surfaces of the film. What physical parameter of the film controls the relative phase? What parameter controls the relative amplitude?

Question 3: (5 points) Use the fact that

\[ e^{i\phi} = \cos \phi + i \sin \phi \quad \text{and} \quad e^{i\alpha} e^{i\beta} = e^{i(\alpha+\beta)} \quad \text{and} \quad i \equiv \sqrt{-1} \]

to derive the expression for \( \sin(\alpha - \beta) \) in terms of \( \sin \alpha, \sin \beta, \cos \alpha, \) and \( \cos \beta. \)
**Question 4:** (5 points) A Fabray-Perot interferometer is fixed to have a mirror separation of \( d = 2 \text{ cm} \). It is enclosed entirely in a chamber filled with air of index of refraction \( n = 1.0003 \) and illuminated by monochromatic light of vacuum wavelength \( \lambda = 600 \text{ nm} \). The chamber is then pumped down to vacuum. How many fringes pass through a point on a screen on the output side near the axis of symmetry during this process? (Recall that the on-axis constructive interference condition is \( 2d = m\lambda \).)

**Question 5:** (5 points) The Arecibo radio telescope has a circular primary mirror diameter of 300 m. What is the smallest size feature that the telescope can resolve on a “nearby” asteroid located \( 5 \times 10^9 \text{ m} \) from Earth using 3 cm radio waves? You may assume that the angular diffraction limit of \( 1.22 \lambda/D \approx \lambda/D \). Give your answer in kilometers.